Determination of Corneal Asphericity after Myopia Surgery with the Excimer Laser: A Mathematical Model

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Purpose. To determine the theoretical change of corneal asphericity within the zone of laser ablation after a conventional myopia treatment, which conforms to Munnerlyn's paraxial formula and in which the initial corneal asphericity is not taken into consideration.

Methods. The preoperative corneal shape in cross section was modeled as a conic section of apical radius R_1 and shape factor p_1. A myopia treatment was simulated, and the equation of the postoperative corneal section within the optical zone was calculated by subtracting the ablation profile conforming to a general equation published by Munnerlyn et al. The apical radius of curvature r_2 of the postoperative profile was calculated analytically. The postoperative corneal shape was fitted by a conic section, with an apical radius equal to r_2 and a shape factor p_2 equal to the value that induced the lowest sum of horizontal residuals and the lowest sum of squared residuals. These calculations were repeated for a range of different dioptric treatments, initial shape factor values, and radii of curvature to determine the change of corneal asphericity within the optical zone of treatment.

Results. Analytical calculation of r_2 showed it to be independent of the initial preoperative shape factor p_1. The determination of p_2 was unambiguous, because the same value induced both the lowest sum of residuals and the lowest sum of the squared residuals. For corneas initially prolate ($p_1 < 1$), prolateness increased ($p_2 < p_1 < 1$), whereas for oblate corneas ($p_1 > 1$), oblateness increased ($p_2 > p_1 > 1$) within the treated zone after myopia treatment. This trend increased with the increasing magnitude of treatment and decreased with increasing initial apical radius of curvature R_1.

Conclusions. After conventional myopic excimer laser treatment conforming to Munnerlyn's paraxial formula, the postoperative theoretical corneal asphericity can be accurately approximated by a best-fit conic section. For initially prolate corneas, there is a discrepancy between the clinically reported topographic trend to oblateness after excimer laser surgery for myopia and the results of these theoretical calculations. (*Invest Ophthalmol Vis Sci.* 2001;42:1736–1742)

In both photorefractive keratectomy (PRK) and laser in situ keratomileusis (LASIK) for myopia, flattening of the central corneal curvature due to tissue photoablation results in decreased refractive power. Although considered proprietary, current excimer laser algorithms rely on the pioneering theoretical work of Munnerlyn et al.[1] They predict the change in corneal power by considering the initial unablated and the final ablated corneal surface as two spherical surfaces, with a single but different radius of curvature.

Given that the corneal surface is aspheric, the corneal shape in cross section can be approximated by a conic section.[2–7] The asphericity of the cornea is then defined by the shape factor of the conic section that approximates it most closely. A high percentage of corneas are prolate.[2,3,7–9] After PRK for myopia, a change from a prolate conformation to an oblate optical contour has been reported.[10–13] To our knowledge, there have been no reports that address the theoretical change in asphericity induced by excimer laser treatment for myopia.

We attempted to predict the theoretical change of corneal asphericity within the optical zone after myopia treatment, conforming with the work of Munnerlyn et al.[1] We developed a mathematical model based on a conic section approximation, enabling prediction of theoretical postoperative asphericity, and investigated the influence of preoperative asphericity, magnitude of correction, and radius of curvature on its outcome.

Materials and Methods

Current excimer laser ablations for treating myopia rely on the pioneering work of Munnerlyn et al.,[1] in which the initial and final corneal surfaces are assumed to be spherical. This allows calculation of the ablation profile by the following general formula (Fig. 1)

$$t(y) = \sqrt{R_2{}^2 - \left(\frac{S}{2}\right)^2} - \sqrt{R_1{}^2 - \left(\frac{S}{2}\right)^2} + \sqrt{R_1{}^2 - y^2} - \sqrt{R_2{}^2 - y^2} \qquad (1)$$

where $t(y)$ expresses the depth of tissue removal as a function of the distance y from the center of an optical zone diameter of S when R_1 and R_2 are the initial and final corneal anterior radii of curvature, respectively. The power of the removed lenticule (D) corresponds to the intended refractive change and is related to R_1, R_2, and the index of refraction (n) as follows

$$D = (n - 1) \cdot \left(\frac{1}{R_1} - \frac{1}{R_2}\right) \qquad (2)$$

where R_2 is more than R_1 for ablations of myopia.

In our theoretical model, we made the following assumptions: The initial corneal surface is rotationally symmetric, and the plane curve of a corneal meridian is a conic section with its apex located at the origin of rectangular x, y coordinates in a system of Cartesian axes.

A conic section can be described mathematically by Baker's equation[14]

$$y^2 = 2Rx - px^2 \qquad (3)$$

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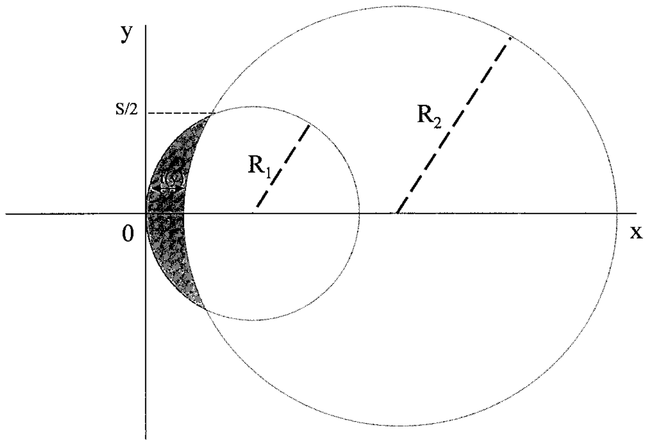

FIGURE 1. Model proposed by Munnerlyn et al.[1] for the ablation profile in the excimer laser treatment of myopia. The investigators predict the change in corneal power by treating the initial unablated and the final ablated corneal surfaces as two spherical diopters, each having a single but different radius of curvature, respectively R_1 and R_2. This model enables the general formula (equation 1) to calculate the ablation profile $t(y)$ as a function of distance to the center of the optical zone of diameter S.

where x and y are the coordinates on a Cartesian system with the axis of revolution, R the apical radius of curvature, and p the shape factor. When p is less than 1, the ellipse is prolate and flattens from the center to the periphery. When p equals 1, the ellipse is a circle. When p is more than 1, the ellipse is oblate and steepens from the center to the periphery.

For our purpose, it is more useful to evaluate x in terms of y. Solving equation 3 for x gives

$$X(y) = \frac{R - \sqrt{(R^2 - py^2)}}{p} \tag{4}$$

When a correction of D diopters is simulated using equation 1 of Munnerlyn et al.[1] on a cornea modeled as a conic section of apical radius R_1 and shape factor p_1, within an optical zone diameter S, the resultant curve X_2 is derived from the following equation

$$X_2(y) = X_1(y) + t(y) \tag{5}$$

$$X_2(y) = \frac{R_1 - \sqrt{(R_1^2 - p_1 y^2)}}{p_1} + \sqrt{R_2^2 - \left(\frac{S}{2}\right)^2} - \sqrt{R_1^2 - \left(\frac{S}{2}\right)^2} + \sqrt{R_1^2 - y^2} - \sqrt{R_2^2 - y^2} \tag{6}$$

This equation does not describe a conic section (Fig. 2). However, the radius of curvature for each point of the curve X_2 is given by $r_2(y)$, which can be computed as follows

$$r_2(y) = \left| \frac{(1 + X_2'(y))^{3/2}}{X_2''(y)} \right| \tag{7}$$

This formula gives the radius of the osculating circle r_2 at any point of the curve (X_2). The first derivative $X_2'(y)$ is

$$X_2'(y) = \left(\frac{y}{\sqrt{R_1^2 - p_1 y^2}} - \frac{y}{\sqrt{-(y^2 - R_1^2)}} + \frac{y}{\sqrt{-(y^2 - R_2^2)}} \right) \tag{8}$$

and the second derivative $X_2''(y)$ is

$$X_2''(y) = \frac{\sqrt{R_1^2 - p_1 y^2} + \frac{p_1 y^2}{\sqrt{R_1^2 - p_1 y^2}}}{(R_1^2 - p_1 y^2)} + \frac{\sqrt{-(y^2 - R_1^2)} + \frac{y^2}{\sqrt{-(y^2 - R_1^2)}}}{(y^2 - R_1^2)} - \frac{\sqrt{-(y^2 - R_2^2)} + \frac{y^2}{\sqrt{-(y^2 - R_2^2)}}}{(y^2 - R_2^2)} \tag{9}$$

After inserting respective first and second derivatives of function $X_2(y)$ into formula $r_2(y)$, the radius of curvature can be calculated by substituting $X_2'(y)$ and $X_2''(y)$ in equation 7. The apical radius of curvature of $X_2(y)$ is $r_2(0)$. It is calculated by substituting 0 for y. When

$$y = 0,\ r_2(0) = R_2 \tag{10}$$

Thus, the radius of curvature of X_2 at the apex (apical radius of curvature) is the same as the final radius of curvature, which is derived from equation 1 (Munnerlyn et al.[1]).

Thus, $X_2(y)$ has an apical radius of curvature R_2, but the shape factor that describes its asphericity cannot be computed by the foregoing calculations, because $X_2(y)$ does not describe a conic section. However, a best-fit conic section, $C_2(y)$, with apical radius of curvature R_2 and shape factor p_2 can be calculated. We plotted multiple conic sections, $C(y)$ with shape factor p_c. Substituting C for X in equation 4 (Fig. 3)

$$C(y) = \frac{R_2 - \sqrt{(R_2^2 - p_c y^2)}}{p_c} \tag{11}$$

To determine the best-fit conic section with shape factor p_2, we used two methods to minimize the sum of the absolute values of the residuals $T(p_c)$ and the sum of the squared residuals $T_s(p_c)$.

We developed a numerical procedure and performed computation on a computer spreadsheet (Excel 97 software; Microsoft, Seattle, WA). We defined 31 points along the hemi y-axis from $y = 0$ to $y = S/2 = 3$ mm, equally spaced by 0.01 mm. For a given R_1, D, and p_1, $T(p_c)$ and $T_s(p_c)$ were iteratively calculated for values of p_c ranging from $(p_1 - 2)$ to $(p_1 + 2)$ by incremental steps of 0.01. Solutions were represented by the value(s) of p_c that induced the smallest $T(p_c)$ and the smallest $T_s(p_c)$. These sums were recorded and tabulated.

Referring to the geometric model of the laser excimer treatment operation for myopia defined herein, we were able to repeat these

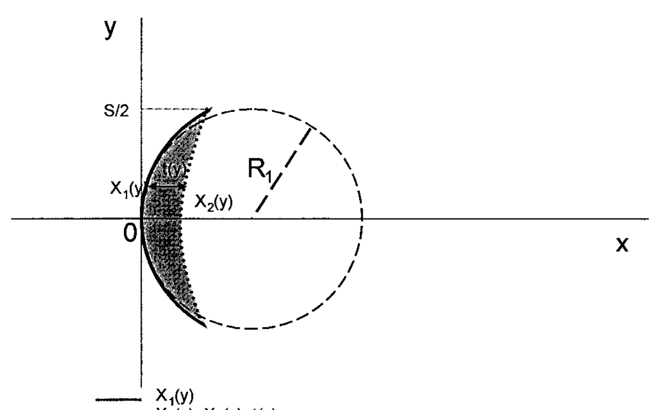

FIGURE 2. Ablation profile for myopia conforming to Munnerlyn's equation applied on an aspheric surface modeled in cross section as a conic section $X_1(y)$ of apical radius equal to R_1. The postoperative cross section $X_2(y)$ is equal to the addition of the initial aspheric profile $X_1(y)$ and the ablation profile $t(y)$.

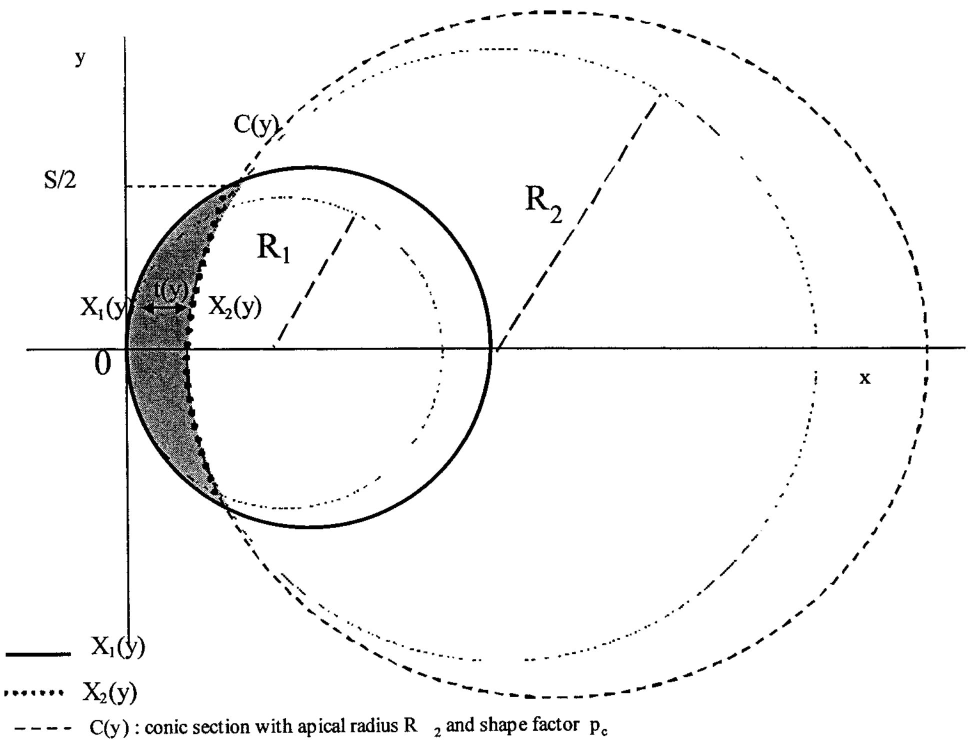

FIGURE 3. Approximation of the postoperative profile $X_{2i}(y)$ by a conic section $C(y)$ of apical radius of curvature R_2 and shape factor p_c.

calculations for a range of different dioptric treatments, initial shape factor values, and radii of curvature. The numerical values of the selected variables are listed in Table 1.

RESULTS

In the theoretical conditions that we used to determine the best-fit conic section, the p_c that corresponded to the lowest $T(p_c)$ was identical with that corresponding to the lowest $T_s(p_c)$ value. The minimal value of $T(p_c)$ and $T_s(p_c)$ corresponding to the same p_c for 31 points thus provided a value for p_2 within our range of testing of ± 0.01. The minimal values of $T(p_c)$ and $T_s(p_c)$ were always less than 2 and 0.1 μm, respectively.

Figure 4 represents the effect of the myopia treatment on initial corneal asphericity. In corneas that were initially prolate ($p_1 < 1$), we found that prolateness increased ($p_2 < p_1 < 1$), whereas in initially oblate corneas ($p_1 > 1$), oblateness increased ($p_2 > p_1 > 1$) within the treated zone after myopia treatment. Spherical corneas remained spherical ($p_2 = p_1 = 1$) after treatment. The slope of asphericity change was not constant but increased with the magnitude of treatment.

Figure 5 shows the effect of the initial radius of curvature on the asphericity outcome after treatment of prolate corneas. For the same treatment parameters, steeper prolate corneas (R = 7.5 mm) tend to become less prolate than flatter prolate corneas (R = 8.1 mm). This effect increases with the magnitude of the treatment. Figure 6 shows the effect of initial radius of curvature in oblate corneas. For the same magnitude of treatment, steeper oblate corneas tend to become less oblate and flatter oblate corneas more oblate.

TABLE 1. Representative Variables within the Normal Range

Initial Radius of Curvature (R_1) (mm)	Initial Asphericity (p_1)	Magnitude of Treatment (Diopters)*
7.5, 7.8, 8.1	0.4, 0.75, 1.0, 1.2, 1.4	−1 to −12

* In 1-diopter steps.

DISCUSSION

Defocus correction is the objective of conventional laser refractive procedures based on the formula of Munnerlyn et al.,[1] which assumes that preoperative corneal surface has a single radius of curvature. The normal human cornea is not spherical, and, despite its shortcomings, modeling the corneal shape in cross section as a conic section is a better approximation and has been widely used[3-6,15] since its introduction by Mandell and St. Helen in 1971.[2] Most human normal corneas conform to a prolate ellipse and flatten from the center to the periphery (negative asphericity; $p < 1$), but some corneas are oblate and steepen from the center to the periphery (positive asphericity; $p > 1$).

Based on topographic observations that the central cornea becomes flatter than the untreated peripheral cornea after excimer laser surgery for myopia, it could be erroneously predicted that the postoperative outcome in all myopic corneas after excimer laser surgery would be increased oblateness. Such a prediction does not take into consideration the changes of corneal curvature within the treatment zone. The focus of our study was to examine the postoperative changes of corneal asphericity within the treatment zone without consideration of the transition zone or the peripheral zone outside the treatment.

We have demonstrated that in prolate and oblate corneas, laser treatments for myopia based on the equation of Munnerlyn et al.[1] result in a final apical radius of curvature that is independent of the initial asphericity (equation 10). Furthermore, we have demonstrated that in prolate corneas these laser treatments result in more prolate configuration within the area of treatment (Fig. 5) and conversely in oblate corneas, the outcome is increased oblateness (Fig. 6).

In initially prolate corneas, there is a discrepancy between the topographic observations and the theoretical predictions

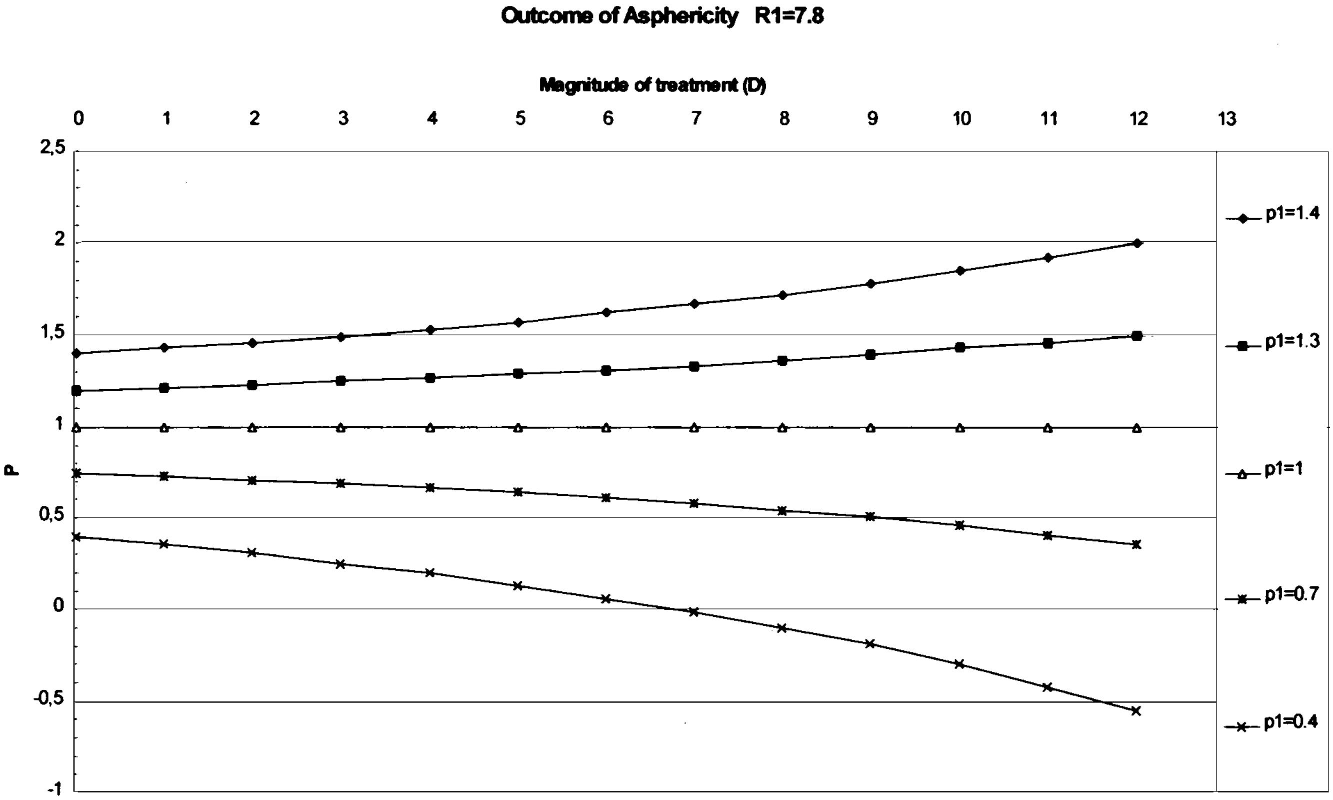

Figure 4. Outcome of asphericity for different magnitudes of correction. The values of p_2 are plotted against the magnitude of the correction for different values of p_1. For corneas that are initially prolate ($p_1 < 1$), prolateness should increase ($p_2 < p_1 < 1$) after myopia treatment, whereas in oblate corneas ($p_1 > 1$), oblateness should increase ($p_2 > p_1 > 1$) within the optical zone after myopia treatment. Spherical corneas remain spherical ($p_1 = p_2 = 1$) after treatment.

of our model with regard to the change in asphericity. This discrepancy may be due to several factors.

First, there are limitations in the elliptical model used in our study. The true corneal section does not conform exactly to an ellipse. The ideal model may have to incorporate additional factors (i.e., higher polynomial) to match the human cornea. The elliptical model is, however, a good approximation of the corneal profile over the central 8 mm of its approximately 12-mm diameter.[16] This area represents the central optical zone, which is flattened after excimer treatment for myopia and through which light passes to form the foveal image. In conventional optics, conic sections are frequently used to model the corneal surface. Furthermore, topographic evaluation of cornea asphericity has been estimated from the conicoid that best fits the keratoscopic or keratometric data.[5–7,17–23]

Our analysis assumes a rotationally symmetric mathematical model, but the human cornea may exhibit toricity. Several groups have investigated the meridional variations of corneal asphericity.[3,7,21] The difference between the maximal and minimal values of asphericity is low, ranging from 0.13 to 0.50 in 80% of the corneas.[8] This difference in the overall asphericity change after laser treatments for myopia does not seem to be significant.

Second, the ablation profile of current lasers for myopia does not follow Munnerlyn's formula. The effects of lasers may differ according to the homogeneity and the location of the laser beam, and laser manufacturers may have altered initial nomograms to improve clinical outcomes and eliminate central islands. Differences in ablation rate of Bowman's membrane and stroma or within the stroma may also contribute to this discrepancy. In addition, the applied fluence at the cornea, even when the laser beam is homogeneous, decreases with the distance from the center because of reflection of the UV light and because of the curvature of the cornea. The achieved ablation pattern could thus differ from the one attempted. However, no published data confirm this possibility. Because the ablation patterns are proprietary, we cannot confidently accept this possibility as the major explanation for the discrepancy.

Third, wound healing (epithelial hyperplasia, stromal remodeling) could be the source of the shape discrepancy. Topography patterns have been shown to change with time.[24] Variations in epithelial thickness and curvature of the epithelial–stromal interface have been implicated in the refractive regression occurring after LASIK and PRK.[25–28] They may modify the specific effect induced by the ablation of myopia and could account for the observed trend to oblateness observed by Hersh et al.,[10] who used corneal topography.

Fourth, photokeratoscopic or videokeratoscopic instruments do not properly assess the shape of the cornea when spherical algorithm assumptions are used.[29] These assumptions have been thought to be responsible for the differences observed between the measured corneal power and the magnitude of change in manifest refraction.[13,30,31] Douthwaite[32] used calibrated convex ellipsoidal surfaces of known apical radius (*R*) and asphericity (*p*) to assess the accuracy of the EyeSys videokeratscope (Premier Laser Systems, Irvine, CA). This device appeared to overestimate both *p* and *R*, especially for asphericities outside the 0.8 to 1.0 region (i.e., the near-spherical zone). In their study of corneal asphericity after PRK, Hersch et al.[10] acknowledge that idiosyncrasies in their Placido-based topography system could have affected their results. These considerations may be important for our pur-

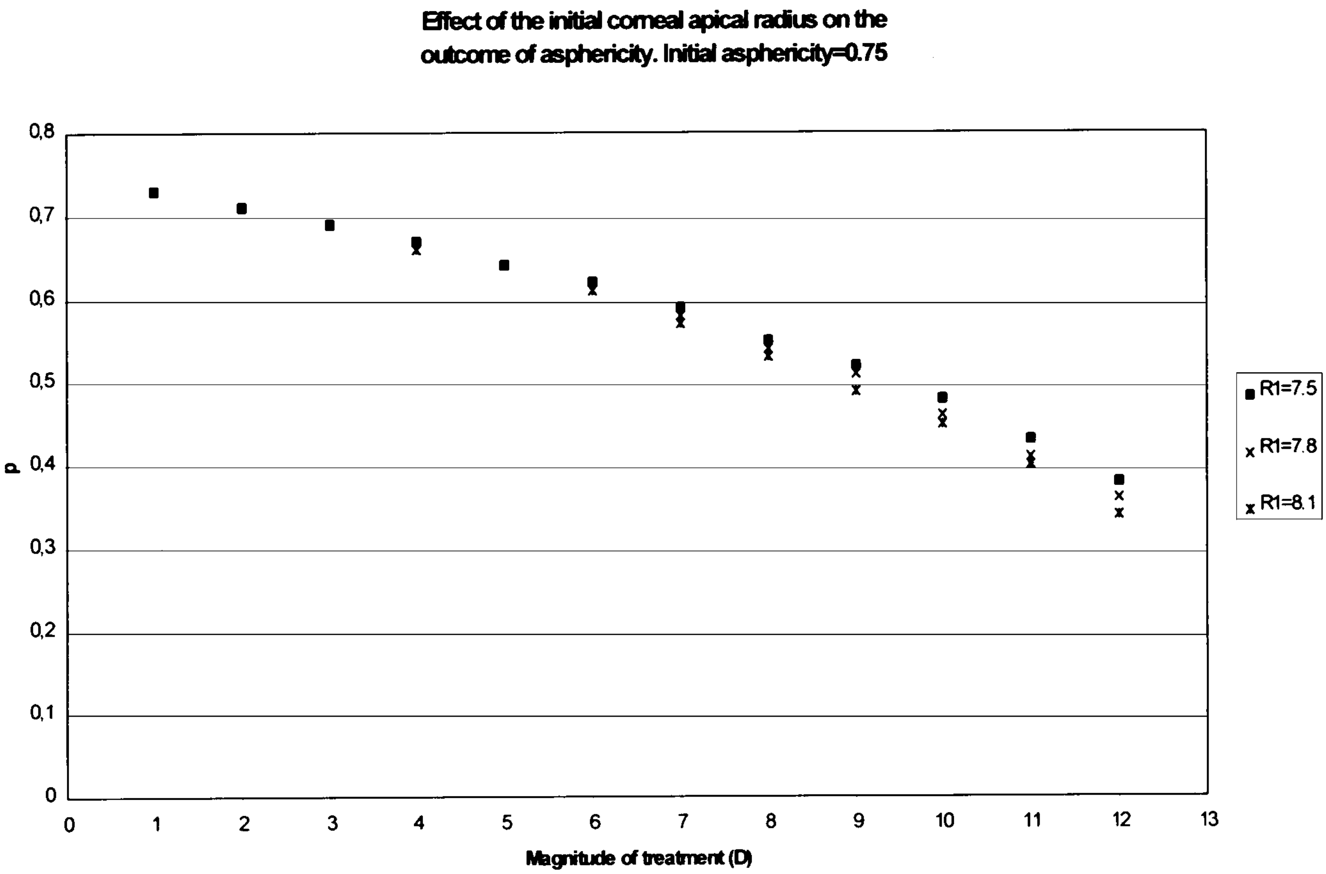

FIGURE 5. Influence of the initial apical radius of curvature on p_2, which is plotted against the magnitude of treatment for three different apical radii: 7.5, 7.8, and 8.1 mm. The initial corneal surface is prolate. Steeper prolate corneas tend to become less prolate and flatter prolate corneas more prolate.

poses. The information provided by keratoscopes after laser refractive surgery is subject to cautious interpretation, and current devices may not be sensitive enough to quantify or assess precisely the postoperative corneal asphericity within the ablated zone.

After the correction of myopia, one of the typical topographical aspects on chromatic maps displays a central circular area of uniform colder color with regard to the surrounding surface. If the overall shape of the cornea after treatment is considered, the central flattening contrasting with the unchanged peripheral contour may lead to the subjective assessment of postoperative oblateness. Our study focused on the determination of the theoretical change of asphericity within the optical zone—that is, the corneal surface of the cornea receiving the laser treatment. Other typical topographical features after PRK have been described, relating nonhomogeneous power within the treated zone.[33] The "keyhole," the semi-circular ablative patterns, and the central islands represent three entities with different clinical issues, but all are characterized by the presence of a higher dioptric power area inside the ablation zone. These features may represent increased prolateness of the cornea. The cause of the central island has not yet been clarified with certainty, although many hypotheses have been offered. Our model suggests that preoperative asphericity could be another factor, especially in patients with preoperative marked prolateness.

These considerations point out the ambiguity in the definition of the asphericity of the corneal surface after refractive surgery for myopia. To clarify the term, provide an accurate baseline description of the corneal profile, and model optical errors such as spherical aberrations, the terms oblate and prolate should refer only to continuous conical shapes.

Our mathematical procedure for finding the best-fitting conic section to our given set of points consisted of minimizing the sum of the offset absolute values and the sum of the square of the offsets. The latter did not allow the residuals to be treated as a continuous differentiable quantity, but may have given to the outlying points a disproportionate effect on the fit, which was not the case. Using two fitting criteria raises the question of which method should have been adopted if there had been discrepancies between the best-fit results of optimizing p_2. The sum of the squared residuals is the most commonly used method. However, we used the sum-of-residual-fitting method to confirm our findings. We found that the determination of p_2 was unambiguous, given the small value of both the sum of the residuals and the sum of the squared residuals. Therefore, the conic section approximation can also be successfully used to describe the corneal profile within the optical zone after a myopia laser treatment conforming to Munnerlyn's equation.

Equations 7 through 10 allow calculation of the apical radius of curvature of the best-fitting conic section to our set of points. This radius is independent of the initial value of the asphericity. This is contradictory to the findings of Patel and Marshall,[34] who found that corneal asphericity could marginally affect the initial refractive outcome of PRK. As in this study, their mathematical model assumed the corneal surface to be a conic section and, for a given correction, the amount of corneal tissue removed by PRK was computed based on spherical corneal optics. However, they arbitrarily assumed that

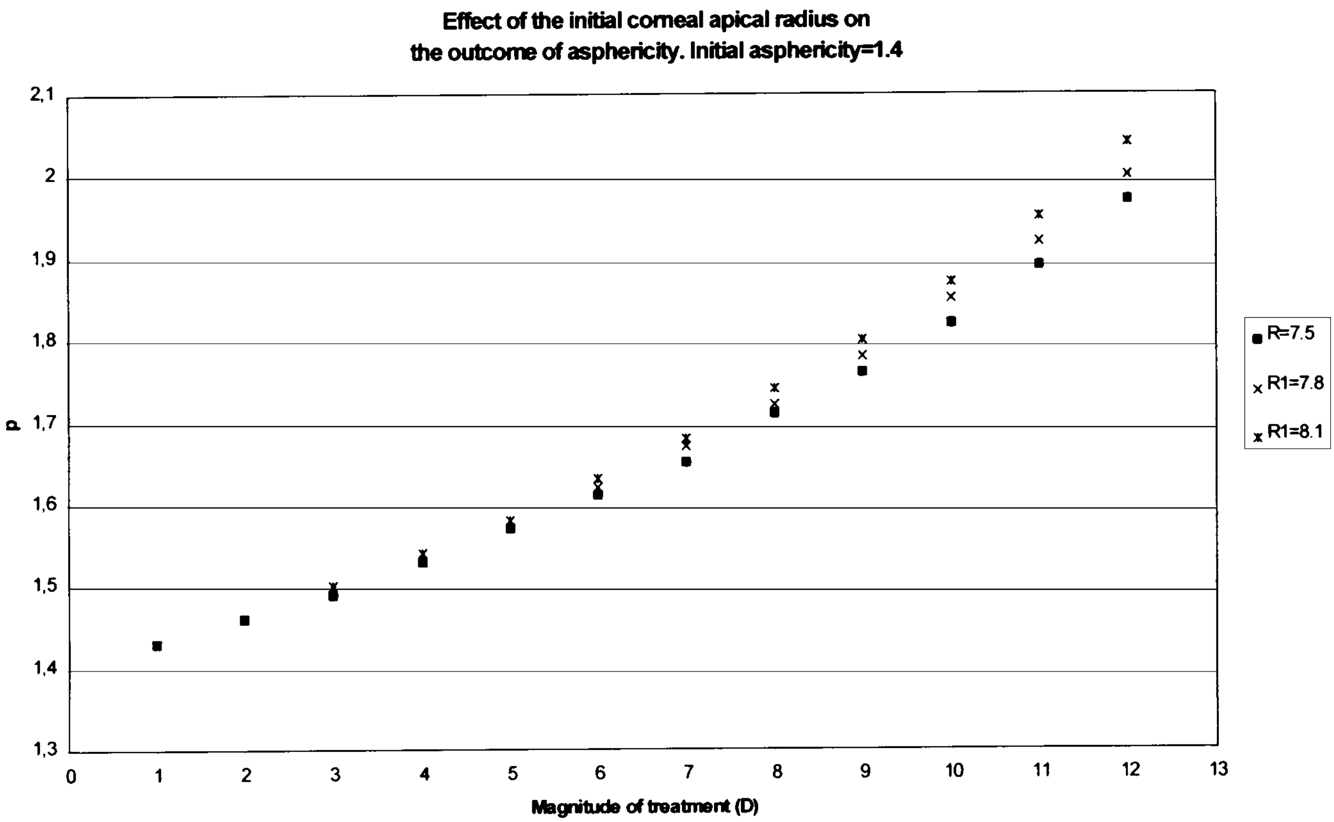

FIGURE 6. Influence for the initial apical radius of curvature on p_2. The initial corneal surface is prolate. Steeper oblate corneas tend to become less oblate and flatter oblate corneas more oblate.

initial and final corneal shapes were typically prolate and oblate, respectively. This assumption led to computation of a different radius of curvature from the one that was expected, according to the spherical model.

There is general agreement that negative corneal asphericity has direct optical significance. It is thought to influence visual performance directly by lowering spherical aberration,[12,16] although this finding remains to be demonstrated. Patel et al.,[17] using optical raytracing of finite schematic eyes, found that the value of p required to eliminate spherical aberration at the anterior surface is -0.528, given a refractive index of 1.376.

Despite the elimination of spherocylindrical errors, refractive surgery may decrease visual performance by altering the corneal shape and inducing unwanted changes in corneal asphericity. Many investigators have noted that after radial keratotomy the cornea becomes oblate, because the paracentral cornea is relatively steeper than the central cornea.[35–38] Seiler et al.[39] proposed an aspheric nomogram for PRK, designed to preserve a negative asphericity. Their clinical results were encouraging, but the aspheric nomogram used did not take into account the patient's preoperative asphericity.

In summary, given that the anterior surface of the human cornea is the main refractive element of the eye, its shape may contribute to optical aberrations. For the normal, untreated, central corneal zone, a conic section can accurately approximate the profile of the corneal zone remodeled by ablation of myopia. To limit or treat optical aberrations after refractive surgery, new profiles including customized aspherical treatments must be developed. The major point of this study—that the modeling of asphericity compares poorly with reported asphericity outcomes—warrants better representation and collection of data before and after treatment. One future rationale may be to use exact raytracing to determine retinal image quality, given a certain ablation profile and a certain shape factor, and then look for ways to alter given asphericities to a desirable level. To reach this goal, further theoretical and clinical studies of corneal shape after laser refractive surgery are needed.

References

1. Munnerlyn CR, Koons SJ, Marshall J. Photorefractive keratectomy: a technique for laser refractive surgery. *J Cataract Refract Surg.* 1988;14:46–52.
2. Mandell RB, St. Helen R. Mathematical model of the corneal contour. *Br J Physiol Opt.* 1971;26:183–197.
3. Kiely PM, Smith G, Carney LG. The mean shape of the human cornea. *Opt Acta.* 1982;8:1027–1040.
4. Bennett AG. Aspherical and continuous curve contact lenses. *Optom Today.* 1988;28:11–14.
5. Douthwaite WA, Sheridan M. The measurement of the corneal ellipse for the contact lens practitioner. *Ophthalmic Physiol Opt.* 1989;9:239–242.
6. Mandell RB. The enigma of the corneal contour. *Contact Lens Assoc Ophthalmol.* 1992;18:267–273.
7. Eghbali F, Yeung KK, Maloney RK. Topographic determination of corneal asphericity and its lack of effect on the refractive outcome of radial keratotomy. *Am J Ophthalmol.* 1995;119:275–280.
8. Douthwaite WA, Hough T, Edwards K, Notay H. The EyeSys videokeratoscopic assessment of apical radius and p-value in the normal human cornea. *Ophthalmic Physiol Opt.* 1999;19:467–474.
9. Hersh PS, Schwartz-Goldstein BH, The Summit Photorefractive Keratectomy Topography Study Group. Corneal topography of phase III excimer laser photorefractive keratectomy. *Ophthalmology.* 1995;102:963–978.
10. Hersh PS, Shah SI, Holladay JT, Summit PRK Topography Study Group. Corneal asphericity following excimer laser photorefractive keratectomy. *Ophthalmic Surg Lasers.* 1996;27(suppl):S421–S428.

11. Maguire LJ, Zabel RW, Parker P, Lindstrom RL. Topography and raytracing analysis of patients with excellent visual acuity 3 months after excimer laser photorefractive keratectomy for myopia. *Refract Corneal Surg.* 1991;7:122–128.
12. Seiler T, Reckmann W, Maloney RK. Effective spherical aberration of the cornea as a quantitative descriptor in corneal topography. *J Cataract Refract Surg.* 1993;19(suppl):155–165.
13. Trocmé SD, Mack KA, Gill KS, Gold DH, Milstein BA. Corneal topography after excimer laser photorefractive keratectomy for myopia. *J Am Optom Assoc.* 1997;68:448–451.
14. Baker TY. Raytracing through non-spherical surfaces. *Proc R Soc.* 1943;55:361–364.
15. Sheridan M, Douthwaite WA. Corneal asphericity and refractive error. *Ophthalmic Physiol Opt.* 1989;9:235–238.
16. Atchison DA, Smith G. *Optics of the Human Eye.* Oxford, UK: Butterworth-Heinemann: 2000;2:11–20.
17. Patel S, Marshall J, Fitzke FW III. Model for predicting the optical performance of the eye in refractive surgery. *Refract Corneal Surg.* 1993;9:366–375.
18. Drasdo N, Fowler CW. Non-linear projection of the retinal image in a wide-angle schematic eye. *Br J Ophthalmol.* 1974;58:709–714.
19. Kooijman AC. Light distribution on the retina of a wide-angle theoretical eye. *J Opt Soc Am.* 1983;73:1544–1550.
20. Liou HL, Brennan NA. Anatomically accurate, finite model eye for optical modeling. *J Opt Soc Am A.* 1997;14:1684–1695.
21. Guillon M, Lydon DP, Wilson C. Corneal topography: a clinical model. *Ophthalmic Physiol Opt.* 1986;6:47–56.
22. Klein SA. A corneal topography algorithm that produces continuous curvature. *Optom Vis Sci.* 1992;69:829–834.
23. Budak K, Khater TT, Friedman NJ, Holladay JT, Koch DD. Evaluation of relationships among refractive and topographic parameters. *J Cataract Refract Surg.* 1999;25:814–820.
24. Simon G, Ren Q, Kervick GN, Parel JM. Optics of the corneal epithelium. *Refract Corneal Surg.* 1993;9:42–50.
25. Wu WCS, Stark WJ, Green WR. Corneal wound healing after193-nm excimer laser keratectomy. *Arch Ophthalmol.* 1991;109:1426–1432.
26. Gauthier CA, Epstein D, Holden BA, et al. Epithelial alterations following photorefractive keratectomy for myopia. *J Refract Surg.* 1995;11:113–118.
27. Fagerholm P, Hamberg-Nyström H, Tengroth B. Wound healing and myopic regression following photorefractive keratectomy. *Acta Ophthalmol.* 1994;72:229–234.
28. Lohmann CP, Güell JL. Regression after LASIK for the treatment of myopia: the role of the corneal epithelium. *Semin Ophthalmol.* 1998;13:79–82.
29. Koch DD, Foulks GN, Moran CT, Wakil JS. The corneal EyeSys system: accuracy analysis and reproducibility of first generation prototype. *Refract Corneal Surg.* 1989;5:424–429.
30. Roberts C. Characterization of the inherent error in a spherically biased corneal topography system in mapping a radially aspheric surface. *J Refract Corneal Surg.* 1994;10:103–116.
31. Applegate RA, Nunez R, Buettner J, Howland HC. How accurately can videokeratographic systems measure surface elevation? *Optom Vis Sci.* 1995;72:785–792.
32. Douthwaite WA. EyeSys corneal topography measurement applied to calibrated ellipsoidal convex surfaces. *Br J Ophthalmol.* 1995;79:797–801.
33. Lin DT, Sutton HF, Berman M. Corneal topography following excimer photorefractive keratectomy for myopia. *J Cataract Refract Surg.* 1993;19:149–154.
34. Patel S, Marshall J. Corneal asphericity and its implications for photorefractive keratectomy: a mathematical model. *J Refract Surg.* 1996;12:347–351.
35. Hemenger RP, Tomlinson A, Caroline PJ. Role of spherical aberration in contrast sensitivity loss with radial keratotomy. *Invest Ophthalmol Vis Sci.* 1989;30:1997–2001.
36. Applegate RA, Howland HC, Buettner J, et al. Corneal aberrations before and after radial keratotomy (RK) calculated from videokeratometric measurements. *Vis Sci Appl Tech Digest Opt Soc Am.* 1994;2:58–61.
37. Schwiegerling J, Greivenkamp JE, Miller JM, et al. The effects of radial keratotomy on the asphericity of the cornea. *Vis Sci Appl Tech Digest Opt Soc Am.* 1996;1:208–211.
38. Fleming JF. Corneal topography and radial keratotomy. *J Refract Surg.* 1986;2:249–254.
39. Seiler T, Genth U, Holschbach A, Derse M. Aspheric photorefractive keratectomy with excimer laser. *Refract Corneal Surg.* 1993;9:166–172.