

Volume Estimation of Excimer Laser Tissue Ablation for Correction of Spherical Myopia and Hyperopia

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PURPOSE. To determine the theoretical volumes of ablation for the laser treatment of spherical refractive errors in myopia and hyperopia.

METHODS. The cornea was modeled as a spherical shell. The ablation profiles for myopia and hyperopia were based on an established paraxial formula. The theoretical volumes of the ablated corneal lenticles for the correction of myopia and hyperopia were calculated by two methods: (1) mathematical approximation based on a simplified geometric model and (2) finite integration. These results were then compared for optical zone diameters of 0.5 to 11.00 mm and for initial radii of curvature of 7.5, 7.8, and 8.1 mm.

RESULTS. Referring to a simplified geometrical model, the volume of ablated corneal tissue was estimated to be proportional to the magnitude of treatment (D) and to the fourth power of the treatment diameter (S^4). For refractive correction of myopia and hyperopia, volume estimations using our formula, $V \cong D \cdot (S/9)^4$, were similar to those obtained by finite integration for optical zone diameters of 0.5 to 8.5 mm and for corneal radii of curvature within the clinical range (7.5, 7.8, and 8.1 mm).

CONCLUSIONS. The theoretical volume of corneal tissue ablated within the optical zone for spherical corrections can be accurately approximated by this simplified formula. This may be helpful in evaluating factors that contribute to corneal ectasia after LASIK for myopia and hyperopia. Treatment diameter (S) is the most important determinant of the volume of tissue ablation during excimer laser surgery. (*Invest Ophthalmol Vis Sci.* 2002;43:1445-1449)

Recent concerns regarding the depth of tissue ablation with the excimer laser during laser in situ keratomileusis (LASIK) raise the general issue of understanding the biomechanical response of the cornea to keratorefractive surgery procedures. Models of the mechanical response of the cornea to the trauma inflicted by both the laser ablation and the flap cut have recently been proposed to explain some of the discrepancies observed between the achieved and intended corrections after refractive surgical procedures.^{1,2} Although the depth of corneal ablation and the thickness of the residual

stromal bed have been implicated as determinants of corneal stability, further studies are necessary to evaluate their exact significance. New models estimating the volume of the corneal tissue ablated by a laser refractive procedure may also be necessary to determine the influence of ablated volumes on corneal stability and the procedure's outcome.

The primary focus of this work is to provide a formula to approximate the volume of tissue ablation for spherical correction in myopia and hyperopia. In this study, we approached the volume calculation by two approaches: finite integration and mathematical approximation. We compared the theoretical values of volumes of tissue removal in the treatment of spherical refractive errors predicted by our geometric approximation with the values given by finite integration.

This work is potentially useful to compare the amount of tissue ablation after primary LASIK and retreatment procedures. It may also contribute to our understanding of the factors that lead to keratectasia after LASIK surgery for myopia as well as factors that limit surgical correction of high hyperopia.

MATERIALS AND METHODS

Calculation of Ablation Profile for Spherical Correction in Myopia and Hyperopia

In both photorefractive keratectomy (PRK) and LASIK for spherical refractive errors, flattening or steepening of the central corneal curvature due to tissue photoablation results in decreased or increased refractive power, respectively. Although the ablation profiles of available excimer lasers are proprietary and may vary from one device to another, conventional ablation profiles for the correction of spherical refractive errors rely on the theoretical pioneering work of Munnerlyn et al.³ in which the corneal surface is assumed to be spherical, and the optical power of the excised lenticule (D) corresponds to the intended change in refraction. Calculation of the ablation profile for the correction of spherical myopia (M) can be performed according to the following general formula (Fig. 1)

$$t_M(y) = \sqrt{R^2 - \left(\frac{S}{2}\right)^2} - \sqrt{R_M^2 - \left(\frac{S}{2}\right)^2} + \sqrt{R_M^2 - y^2} - \sqrt{R^2 - y^2} \quad (1)$$

with

$$D = (n - 1) \cdot \left(\frac{1}{R} - \frac{1}{R_M} \right) \quad (2)$$

where R and R_M are the initial and final anterior radii of curvature, respectively ($R > R_M$), and n is the refractive index of the cornea. $t_M(y)$ expresses the depth of tissue removal in treating myopia as a function of the distance y from the center of the treatment with an optical zone diameter of S (in millimeters).

The maximal depth of ablation (t_o) occurs at the center of the treatment zone ($y = 0$) and is calculated by

$$t_o = \sqrt{R^2 - \left(\frac{S}{2}\right)^2} - \sqrt{R_M^2 - \left(\frac{S}{2}\right)^2} \quad (3)$$

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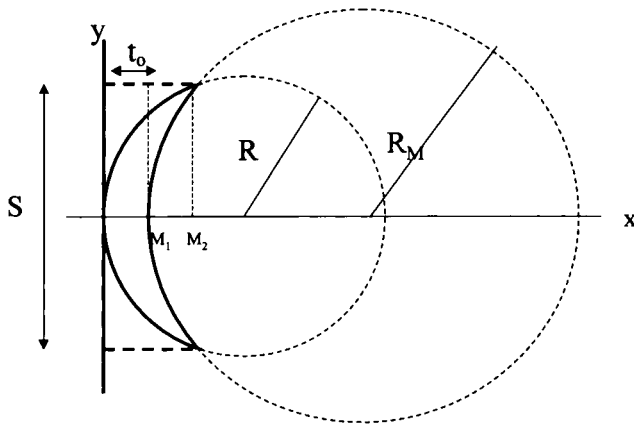


FIGURE 1. Diagram of laser ablation, for myopia, of a spherical surface with initial radius of curvature R and final radius of curvature R_M ($R_M > R$). The depth of ablation $t_M(y)$ is the distance between the initial and final surfaces (equation 1). It varies between a maximum t_o at $y = 0$ and zero at $y = S/2$. Along the x -axis, M_1 corresponds to the maximal depth of tissue ablation and M_2 corresponds to interception of the chord joining the edges of the treatment zone.

To correct hyperopia (H), the surface has to be steepened over the treatment zone (Fig. 2). The ablation profile for the correction of spherical hyperopia is

$$t_H(y) = \sqrt{R^2 - y^2} - \sqrt{R_H^2 - y^2} + R_H - R \quad (4)$$

where $t_H(y)$ expresses the depth of tissue removal in treatment of hyperopia as a function of the distance y from the center of the treatment zone (of diameter S), and R and R_H are the initial and final corneal anterior radii of curvature, respectively.

The maximal depth of ablation occurs at the external limit of the optical zone and is determined by

$$t_o = \sqrt{R^2 - \left(\frac{S}{2}\right)^2} - \sqrt{R_H^2 - \left(\frac{S}{2}\right)^2} + R_H - R \quad (5)$$

Mathematical Approximation of the Volume of Tissue Ablated for Spherical Myopic Corrections

The simplified equation used to estimate the maximal depth of the ablation is³

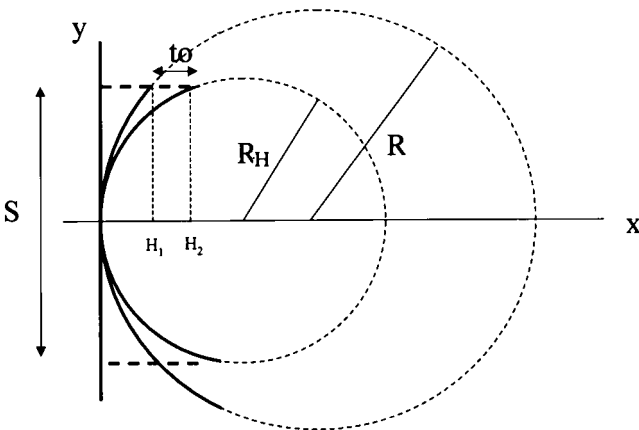


FIGURE 2. Diagram of laser ablation for hyperopia. The initial radius of curvature R is greater than the final radius of curvature R_H . The depth of ablation increases from zero at $y = 0$ to t_o at $y = S/2$. Along the x -axis, H_1 and H_2 are the intercepts of the chords formed at the treatment edges of the untreated and treated corneal surfaces, respectively.

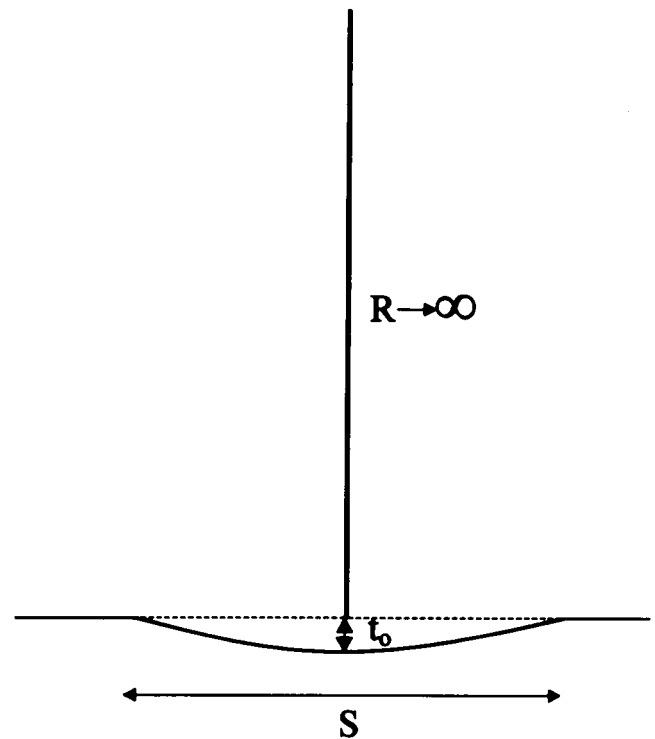


FIGURE 3. Schematic diagram of laser ablation of depth t_o and diameter S , corresponding to a myopia correction of D (diopters), is applied to a flat surface ($R = \infty$). The volume of the cap of ablation is estimated from equation 7 to be $D(S/9)^4$, as in equation 11.

$$t_o \approx \frac{1}{8(n-1)} S^2 D \quad (6)$$

An initially flat surface has a radius of curvature infinitely high and no optical power. The ablated volume needed to induce a power of D diopters over an optical surface of S millimeters is assimilated as a spherical cap of height t_o . The result of a spherical ablation for myopia on a theoretical flat corneal surface would thus be the sculpting of a crater with a shape and volume equal to those of a spherical cap.

Figure 3 represents the cap in cross section. The diameter of the cap is equal to the diameter of the treatment zone (S), and its height corresponds to the maximal depth of ablation t_o calculated by equation 6. Thus, the volume of the spherical cap can be derived at by the following formula:

$$V_{\text{cap}} = \frac{1}{6} \pi t_o \left[3 \left(\frac{S}{2} \right)^2 + t_o^2 \right] \quad (7)$$

Because t_o^2 is very much lower than S^2 , t_o^2 can be neglected, and the volume of the cap can be approximated by

$$V_{\text{cap}} \approx \frac{1}{6} \pi t_o 3 \left(\frac{S}{2} \right)^2 \approx \frac{1}{8} \pi t_o S^2 \quad (8)$$

Replacing t_o by its expression as a function of D and S defined in equation 6 yields

$$V_{\text{cap}} \approx \frac{1}{64,000(n-1)} \pi D S^4 \quad (9)$$

(in cubic millimeters). Since

$$\frac{\pi}{64,000(n-1)} \approx \left(\frac{1}{9} \right)^4 \quad (10)$$

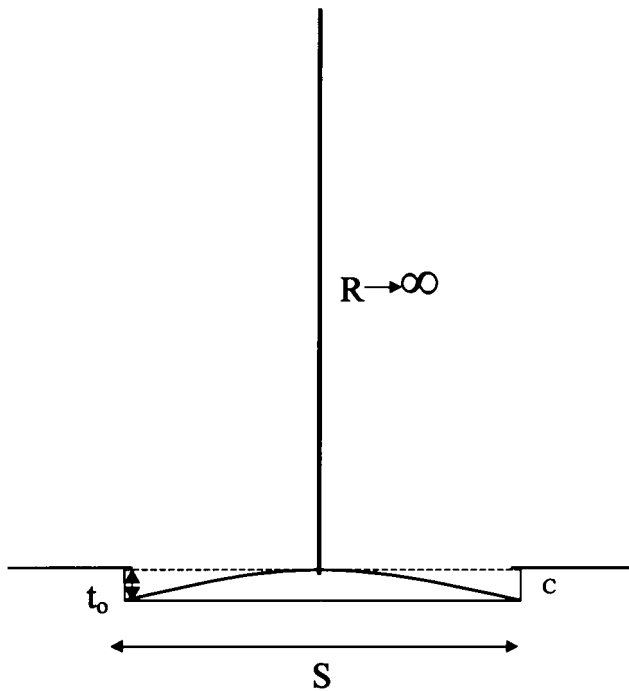


FIGURE 4. Schematic diagram of laser ablation, for hyperopia, of optical zone S and peripheral depth t_o . The treatment is applied to a flat surface to allow approximation of the volume of ablation.

for a myopia treatment of D diopters, the following is used

$$V_M \approx D \left(\frac{S}{9} \right)^4 \quad (11)$$

where V_M is the approximate volume of tissue ablated for spherical correction in myopia (in cubic millimeters), D is the intended change in myopia (in diopters), and S is the diameter of the treatment zone (in millimeters).

Mathematical Approximation of the Volume of Tissue Ablated for Spherical Correction in Hyperopia

A spherical ablation for hyperopia on a theoretical flat corneal surface would result in the sculpting of a dome, centered by the annular ablation zone of increasing depth from the center to the periphery (Fig. 4). The maximal depth of ablation t_o is determined by equation 6, and the volume of this dome can be calculated by using the formula for the volume of a spherical cap. The volume of ablated tissue V_H is obtained by subtracting the volume of the dome (which is equal to the volume of a spherical cap of height t_o and diameter S) from the volume of the corresponding cylindrical segment V_C .

The volume of the cylinder segment is

$$V_C = \pi \left(\frac{S}{2} \right)^2 t_o. \quad (12)$$

Using equation 7 for the volume of the cap and equation 12

$$V_H \approx \pi \left(\frac{S}{2} \right)^2 t_o - \frac{1}{8} \pi t_o S^2 \approx \frac{1}{8} \pi t_o S^2 \approx D \left(\frac{S}{9} \right)^4 \quad (13)$$

This leads to the same approximation as for the spherical correction in myopia (equation 11).

Volume Calculation of Spherical Ablation in Myopia by Finite Integration

Figure 1 represents in cross section the ablated lenticule for correction of myopia. Because of the symmetry around the x -axis, the lenticule of corneal tissue ablated can be calculated using the general formula for the volume of solids of rotation and, by using mathematical integration, the volume V_M of the myopic ablated lenticule can be computed

$$\begin{aligned} V_M &= \pi \int_0^{M1} (2Rx - x^2) dx - \pi \int_{M2}^{M1} (2R_M x - x^2) dx \\ &= \pi \left[(R - R_M) M_1^2 + M_2^2 \left(R_M - \frac{M_2}{3} \right) \right] \end{aligned} \quad (14)$$

with

$$M_1 = R - \sqrt{R^2 - \left(\frac{S}{2} \right)^2} \quad (15)$$

and

$$M_2 = R_M - \sqrt{R_M^2 - \left(\frac{S}{2} \right)^2} \quad (16)$$

where R is the radius of the initial corneal surface, R_M is the radius of the final corneal surface, and S is the optical zone diameter. M_1 corresponds to the maximal depth of tissue ablation, and M_2 is the length of the line segment from the apex of the postoperative anterior surface to the chord formed at the treatment zone diameter (Fig. 1).

Volume Calculation of Spherical Ablation in Hyperopia by Finite Integration

Figure 2 represents the ablated lenticule for correction of hyperopia. The equation to calculate the hyperopic lenticule's volume by finite integration is

$$\begin{aligned} V_H &= \pi \int_0^{H1} (2Rx - x^2) dx - \pi (H_2 - H_1) \left(\frac{S}{2} \right)^2 - \pi \int_{H1}^{H2} (2R_H x - x^2) dx \\ &= \pi \left[(R - R_H) H_1^2 - R_H H_2^2 - \frac{H_2^3}{3} - (H_2 - H_1) \left(\frac{S}{2} \right)^2 \right] \end{aligned} \quad (17)$$

with

$$H_1 = R - \sqrt{R^2 - \left(\frac{S}{2} \right)^2} \quad (18)$$

and

$$H_2 = R_H - \sqrt{R_H^2 - \left(\frac{S}{2} \right)^2} \quad (19)$$

where R is the radius of the initial corneal surface, R_H is the radius of the final corneal surface, S is the optical zone diameter, and H_1 and H_2 are the lengths of the line segment from the anterior surface apex to the chord formed at the treatment zone diameter of the initial and final corneal surfaces, respectively (Fig. 2).

The values provided by these formulas were analyzed and compared for treatment zone diameter(s) ranging from 0.5 to 11.0 mm and for initial radii of curvature within the clinical range for normal corneas

Initial radius=7.8 mm, Optical zone diameter =6 mm

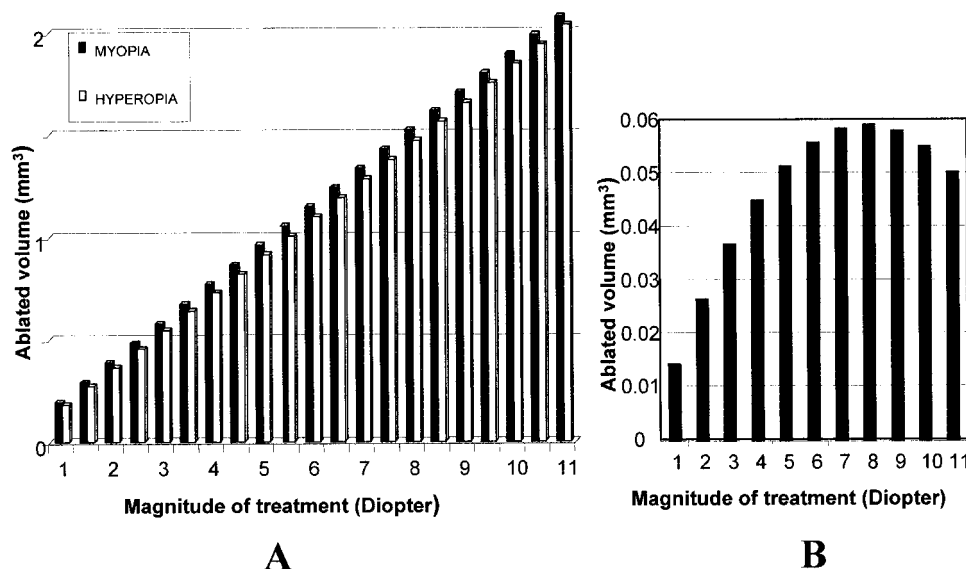


FIGURE 5. Volume comparison between myopic and hyperopic lenticules (A). The difference in the volume of ablated tissue between corrections for myopia and hyperopia is greatest for 8-D corrections (B).

(R of 7.5, 7.8, and 8.1 mm). The refractive index of the stroma (n) was set at 1.377.

RESULTS

Figure 5 shows the distribution of the volumes of ablated tissue, calculated by finite integration, for various magnitudes of spherical correction in myopia and hyperopia and for initial radius of curvature (R) of 7.8 mm and optical zone (S) of 6.0 mm. For similar magnitude of treatments, the ablation volume necessary for spherical correct in myopia is slightly greater than that for spherical correction in hyperopia.

The numeric results of comparison of the theoretical values predicted by actual and approximated calculations of the volumes of ablated tissue for initial radius of curvature (R) of 7.8 mm and optical zone diameter (S) of 4 to 8 mm are shown in Table 1. Volume approximations were within $\pm 2.0\%$ in all calculations for hyperopia and myopia with S less than 7 mm. Similarly, Figure 6 illustrates the accuracy of our formulas for volume approximation for myopia and hyperopia given by equation 11 for R of 7.5, 7.8, and 8.1 mm. Comparison of the ablated corneal volumes, as determined by the finite integration method, with the volume approximations of equation 11, shows that our formula provides an acceptable estimate of the ablated volume for spherical myopia and hyperopia for optical zone diameters up to 9 mm. Beyond 9 mm ($S \geq 9.5$ mm), our

formula tends to underestimate the ablated volume compared with the finite integration calculations, especially for treatment of myopia. As expected, the higher the initial radius of curvature (i.e., the flatter the initial corneal surface), the better is the approximation provided by our formula.

DISCUSSION

We have derived a simple formula that allows estimation of the amount of tissue removed by excimer laser photoablation to correct spherical refractive errors. The volume of photoablated tissue is a function of the magnitude of the treatment and the optical zone diameter to the fourth power, both of which may represent risk factors for keratectasia.

To our knowledge, no studies have been conducted to examine the question of the calculation of the volume of corneal tissue ablated in excimer laser refractive surgery. The surface area of the cornea, however, has been approximated to be 120 ± 2.2 mm² in normal and keratoconus corneas, suggesting that keratoconus is not true ectasia, in which the total surface area increases, but rather is a specialized type of warp-⁴ No similar investigation has been undertaken of keratectasia after refractive lamellar surgery.

Equations 14 to 19 provide methods of volume calculation that are more accurate than the approximations provided in equations 11 and 13. In building our approximations on equation 6, we have limited the accuracy to treatment diameters of up to 7.0 mm. This is not surprising, given that the approximation for calculating the depth of tissue removal in the center of the treated area (equation 6) is tolerably accurate only for small treatment diameters. However, the value of making the approximations of equation 11 and 13 is not only in the simplicity of these equations (e.g., $(V \approx D(S/9)^4)$) but also in the impact of the equations on underscoring the potential danger of increasing treatment diameter in relation to an excessive volume of tissue ablation.

The occurrence of keratectasia after LASIK has raised the need for understanding the biomechanics of the cornea. Mechanisms different from those of keratoconus may explain iatrogenic ectasia after LASIK surgery, given the tissue ablation and the presence of a flap during LASIK. It is believed that the residual stromal bed may be the critical factor in corneal stability in LASIK and other lamellar refractive surgical proce-

TABLE 1. Finite Integration and Approximation Volumes of Ablated Lenticules for Correction of Hyperopia and Myopia

Optical Zone Diameter (mm)	Finite Integration (Hyperopia)	Volume Approximation	Finite Integration (Myopia)
4	0.0345	0.039	0.0356
4.5	0.0558	0.0625	0.0581
5	0.0859	0.0953	0.0904
5.5	0.1274	0.1395	0.1355
6	0.1828	0.1975	0.197
6.5	0.2556	0.2721	0.2794
7	0.3496	0.366	0.3883
7.5	0.4692	0.4823	0.5304
8	0.6198	0.6243	0.7144

Data are expressed as cubic millimeters per diopter of correction.

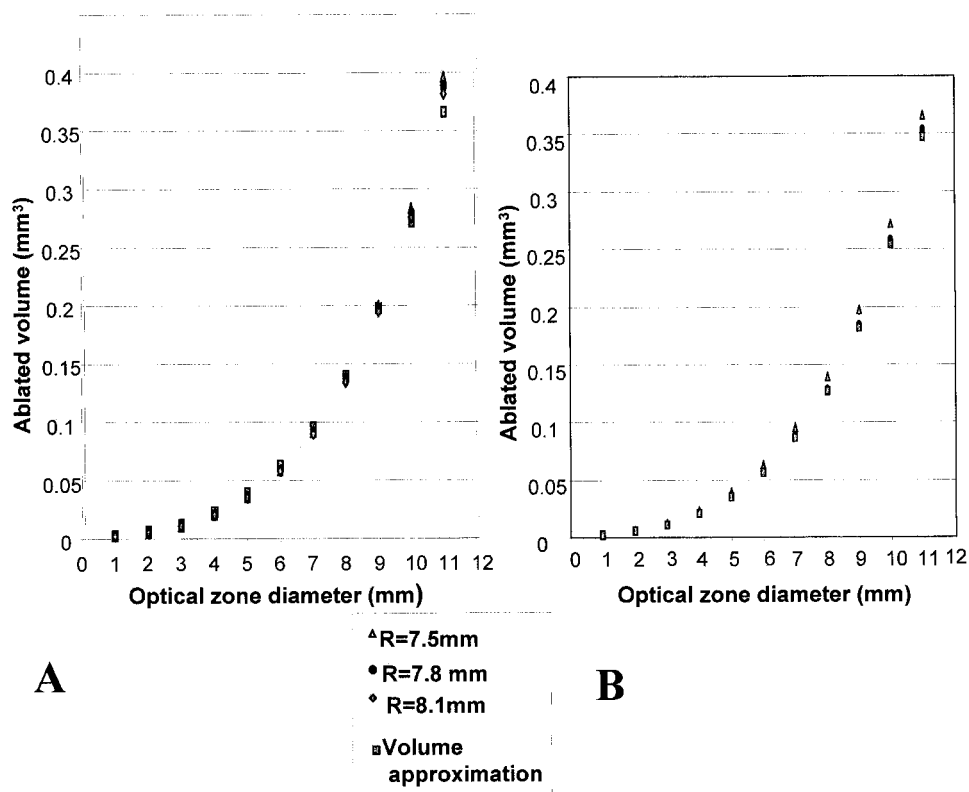


FIGURE 6. (A) Comparison between the values predicted by approximation and finite integration for different corneal radii of curvature, for the treatment of 1 D of spherical myopia. (B) Comparison between the value predicted by approximation and finite integration for different corneal radii of curvature, for the treatment of 1 D of spherical hyperopia.

dures.^{5,6} Geggel and Talley⁷ have reported a case of keratectasia that occurred after 6.6 D of myopia correction with an estimated posterior corneal thickness of 160 μ m. No evidence of forme fruste keratoconus or unusually thin cornea was noted to explain the occurrence of this complication. Based on retrospective analysis of similar cases of corneal ectasia after high magnitudes of myopia treatment, several safety guidelines have been proposed, including leaving a minimal residual stromal bed thickness of 250 μ m and at least 50% of the initial corneal thickness.

Changes in posterior corneal curvature were also reported after uncomplicated LASIK^{6,8} and PRK,⁹⁻¹¹ in which the 250- μ m rule was not violated, suggesting that factors other than residual bed thickness could play a causative role. Argento et al.¹⁰ and Pallikaris et al.¹¹ have postulated that the amount of tissue removed may be another factor influencing the development of ectasia. The use of equations 11, 13, 14, and 17 in similar studies will be valuable in determining the specific situations in which the volume of ablated tissue is a major contributing factor to keratectasia after LASIK surgery.

It is interesting to note that the volume of ablation to correct hyperopia, in the absence of a transition zone, is very similar to that of corresponding diopters of myopia. It is likely that the added volume of tissue ablation used to create the large hyperopic transition zones in hyperopia leads to ectasia of the peripheral cornea, which flattens the central cornea, and may contribute to the limited ability of LASIK surgery to correct high degrees of hyperopia.

Our data suggest that for a given patient with a given flap thickness and a given amount of intended myopia correction, the diameter of the optical zone may be the most important variable influencing long-term corneal stability after LASIK. Given that the volume of ablation is proportional to the fourth power of the treatment diameter, in patients who have large pupils and high myopia, the advantage of a large-diameter

treatment would be outweighed, not only by the greater depth of ablation (proportional to the square of the diameter) but also by the greater increase in the volume of tissue ablation (which is proportional to the fourth power of the diameter).

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