Effect of Anterior Corneal Surface Asphericity Modification on Fourth-Order Zernike Spherical Aberrations

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ABSTRACT

PURPOSE: To evaluate the theoretical influence of the change in corneal asphericity (ΔQ) on the change in fourth-order Zernike spherical aberration coefficient (ΔC_{40}) with customized aspheric refractive correction of myopia and hyperopia.

METHODS: The initial anterior corneal surface profile was modeled as a conic section of apical radius of curvature R_0 and asphericity Q_0. The postoperative corneal profile was modeled as a conic section of apical curvature R_1 and asphericity Q_1, where R_1 was computed from defocus D, and Q_1 selected for controlling the postoperative asphericity. The corresponding change in fourth-order spherical aberration (ΔC^0_4) was computed within a 6-mm optical zone using inner products applied to the incurred optical path changes. These calculations were repeated for different values of D, R_0, Q_0, and various intended ΔC^4_0 values.

RESULTS: Increasing negative spherical aberration ($\Delta C^4_0 < 0$) requires a change toward more negative values of asphericity (increased prolateness; $\Delta Q < 0$) for hyperopic and low myopic corrections, but more positive values ($\Delta Q < 0$) for high myopic correction. The larger the intended change in corneal spherical aberration (ΔC^4_0), the more myopic the threshold value for which the required change in asphericity, ΔQ, becomes positive. The influence of the magnitude of paraxial defocus correction is less pronounced when larger changes in C^4_0 are intended.

CONCLUSIONS: These results provide a basis for controlling the direction (sign) and the magnitude of spherical aberration changes when using customized aspheric profiles of ablation.

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Spherical aberration is a rotationally symmetric aberration in which the effective power of the system depends on the ray height in the pupil. It creates a relative excess or diminution of peripheral versus paraxial refractive power. Re-shaping the corneal contour by excimer laser photoablation and modifying the corneal asphericity was initially proposed to reduce the total ocular spherical aberrations while correcting the myopic or hyperopic defocus.[1,2] Increasing corneal prolateness induces negative spherical aberration, which can result in substantial reduction of the total ocular spherical aberrations. Induction of negative spherical aberrations in an optical system, including the human eye, may also serve to extend the depth of focus and through-focus visual acuity,[3-7] but this is achieved at the cost of deteriorating contrast sensitivity.[8] The increased depth of field resulting from the induction of negative spherical aberration for the whole eye may make it possible for a target to move closer to the unaccommodating eye with a relatively small decrease of visual quality. Thus, modifying the corneal asphericity by creating a hyperprolate corneal surface has been proposed as a surgical approach to compensate for presbyopia.[9-14]

The effect of change in corneal asphericity on the corneal spherical aberration has been examined theoretically[15-17] and experimentally,[18] and correlations between corneal asphericity and spherical aberration have been studied before and after refractive surgery.[19-21] The efficacy of customized Q-value ablation in presbyopic ametropes requires correction of the associated refractive error and adjustment of the desired postoperative asphericity (Q-value) to match the intended change in spherical aberration of corneal origin. However, parame-

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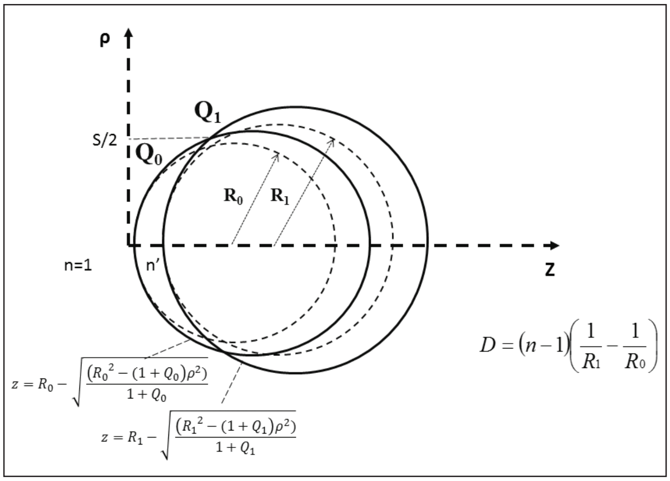

Figure 1. Determination of the aspheric profile of ablation for myopia correction, and control of the asphericity. R_0 and R_1 and Q_0 and Q_1 correspond to the apical radii and asphericities of the preoperative and postoperative corneal surfaces, respectively (μm is a radial coordinate).

ters such as the level of defocus correction (which incur the modification of the apical curvature of the corneal surface) also influence the net change in corneal spherical aberration.[22,23] The theoretical relationships between corneal asphericity change accompanying the surgical correction of the refractive error and the variation of the corneal spherical aberration have not been extensively examined.

In the current study, we sought to determine the effect of the change in corneal asphericity and apical curvature on corneal spherical aberrations for various magnitudes of intended hyperopic and myopic corrections. Although the modification of the corneal asphericity would induce a modification of all rotationally symmetrical Zernike components,[20] we limit our analyses to the fourth-order Zernike spherical aberration term ($Z^4{}_0$), which is the most clinically relevant term. The impact of the change in corneal asphericity and apical radius of curvature on lower- (eg, second order) and higher-order (eg, sixth order) Zernike terms will be examined in a separate study.

MATERIALS AND METHODS

Corneal Profile and Zernike Expansion

In this study, the strength of spherical aberration is quantified by the Zernike coefficient, $C^4{}_0$, and the corresponding Zernike polynomial, $Z^4{}_0$, for the wave aberration associated with the anterior corneal surface for points objected located on the axis at infinity.

We modeled a preoperative corneal surface profile as a conic section of apical radius R_0 and asphericity Q_0, and a postoperative conic section of apical radius R_1 and asphericity Q_1. The theoretical value of the postoperative apical radius of curvature, R_1, was computed using a paraxial formula from the value of R_0 and the distance defocus, D, at the corneal plane (**Figure 1**).

By using scalar products, the approximation of the rotationally symmetrical corneal surface, whose profile is a conic section, can be converted in a rotationally invariant Zernike polynomial expansion (m = 0) over a zone of diameter S. This allows us to obtain the values of the coefficients $C^4{}_0i$ and $C^4{}_0f$ of the $Z^4{}_0$ polynomials corresponding to the preoperative and postoperative corneal profiles, respectively.

Variation of the Corneal Spherical Aberration

The variation in the corneal spherical aberration, caused by the change in the value of the apical radius and asphericity of the corneal profile, was computed because the difference between the final postoperative, $C^4{}_0f$, and the initial preoperative, $C^4{}_0i$, values multiplied by the theoretical change in the refractive index from the air (n = 1) to the corneal stroma (n = 1.376)[24]: $\Delta C^4{}_0 = (C^4{}_0f - C^4{}_0i) \times 0.376$.

The required change of asphericity ΔQ to achieve a variation $\Delta C^4{}_0$ with a customized correction of defocus was approximated as described in **Appendix A** (available in the online version of the article).

This allowed us to calculate the theoretical value of ΔQ to achieve the desired change in spherical aberration ($\Delta C^4{}_0$), based on the values of preoperative corneal curvature, asphericity, and required defocus correction (R_0, Q_0, and D_0). The calculations were performed for a 6-mm ablation zone at the corneal plane.

RESULTS

Determination of Intended Change in the Q Value ($\Delta Q = Q_1 - Q_0$) for Achieving a Null ($\Delta C^4{}_0 = 0$) or Negative Change of Corneal Spherical Aberration ($\Delta C^4{}_0 < 0$) for Various Magnitudes of Defocus Correction

Figure 2 plots the values of the required variation in asphericity (ΔQ) to change the corneal spherical aberration by different amounts ($\Delta C^4{}_0$ = 0, -0.1, -0.2, -0.3, and -0.4 µm) for corrections comprised between -8 and +6 diopters (D) delivered on a corneal profile of 7.8-mm apical radius and prolate preoperative asphericity (Q = -0.2).

When the intended change in spherical aberration is null ($C^4{}_0f = C^4{}_0i$; $\Delta C^4{}_0 = 0$), the change in the corneal asphericity is of opposite sign for myopic versus hyperopic corrections. For myopic corrections, the required change in the corneal asphericity to maintain the corneal spherical aberration unchanged ($\Delta C^4{}_0 = 0$) is positive (ie, decreased prolateness and increased oblateness). For hyperopic corrections, to satisfy the

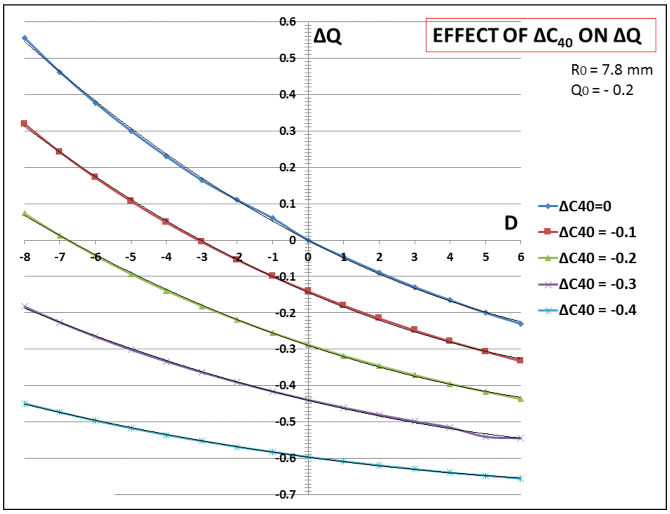

Figure 2. Effect of intended change of spherical aberration on the required change in anterior corneal asphericity ΔQ (x axis) for corrections comprised between -8 and +6 diopters (D) by 1-D steps (y axis).

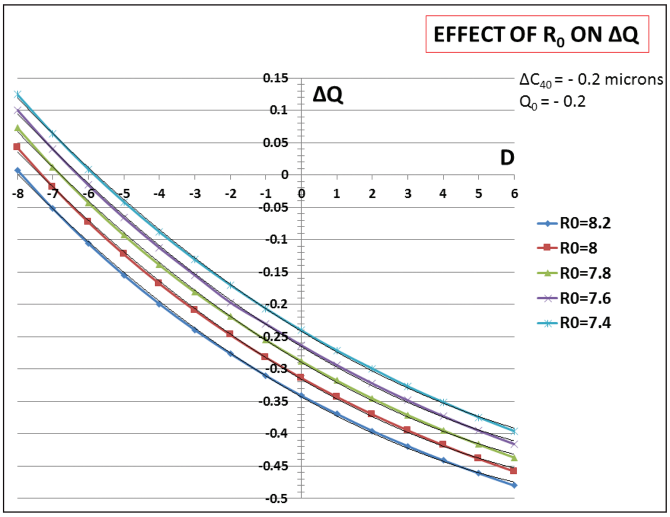

Figure 3. Effect of the apical radius of curvature (R_0) on the required change in the corneal anterior asphericity ΔQ (x axis) to modify the spherical aberration by -0.2 μm on a 6-mm optical zone for corrections comprised between -8 and +6 diopters (D) (y axis).

same condition (ΔC^4_0 = 0), the change in the corneal asphericity is negative (increased prolateness and decreased oblateness).

For prolate preoperative asphericity Q = -0.2, ΔQ ranges between +0.56 (for a myopic correction of -8 D) and -0.23 (for a hyperopic correction of +6 D).

The larger the intended change in corneal spherical aberration (ΔC^4_0), the more myopic the threshold value for which the required change in asphericity becomes positive.

For an intended ΔC^4_0 = -0.1 µm, the change in Q value (ΔQ) is positive for corrections ranging between D = -8 and -3 D (less prolateness), and negative (more prolateness) for lower degrees of preoperative myopia and for preoperative hyperopia.

For an intended ΔC^4_0 = -0.2 µm, the change in Q value is positive for corrections between -8 and -7 D, and for lower degrees of preoperative myopia and for preoperative hyperopia.

For an intended ΔC^4_0 = -0.3 or -0.4 µm, the required change is an increase in the corneal prolateness for all degrees of preoperative myopia and hyperopia.

The influence of the magnitude of paraxial defocus correction tends to be less pronounced when larger changes in C^4_0 are intended. To maintain the preoperative level of spherical aberration (ΔC^4_0 = 0), a correction of -8 D would require the change in asphericity to be in the oblate direction (ΔQ = +0.56), whereas a correction of +6 D would require a change toward increased prolateness (ΔQ = -0.23); hence, the amplitude of the change in asphericity is 0.79. To reduce the spherical aberration by 0.4 µm, the amplitude of change in ΔQ is 0.20 (ranging from ΔQ = -0.45 for a -8 D correction to ΔQ = 0.65 for a +6 D correction).

Effect of the Apical Radius of Curvature (R_0) on the Target Q Value

For the same expected reduction of the corneal spherical aberration of ΔC^4_0 = -0.2 µm (Q_0 = -0.2), the increase in apical curvature (steep cornea; decrease in the value of R_0) results in a slight variation in the magnitude of the predicted change in asphericity, ΔQ (**Figure 3**). The flatter the cornea, the less oblate (or the more prolate) the predicted variation in asphericity. The effect of the preoperative apical radius seems weaker than that of the preoperative corneal asphericity. Changing the apical radius of curvature from 7.4 to 8.2 mm would result in a variation of the required ΔQ from 0.12 to 0 for a correction of -8 D, and from -0.40 to -0.48 for a correction of +6 D.

Effect of the Preoperative Asphericity (Q_0) on the Target Q Value

Figure 4 plots the change in asphericity (ΔQ) required to reduce the spherical aberration by -0.2 µm (ΔC^4_0 = -0.2 µm, R_0 = 7.8 mm) for various corrections and values of corneal asphericity (Q values ranging from +0.1 to -0.4). The more prolate the preoperative surface curvature, the lower the amplitude of the required variation in corneal asphericity ΔQ for the -8 to +6 D correction interval. The threshold value for which the required change in asphericity ΔQ is positive (increased oblateness) occurs for various myopic corrections magnitude, which is higher (more myopic defocus correction) when the cornea is initially more prolate. For a correction of +1 D, the influence of the initial corneal asphericity seems non-significant, because the curves intersect at a same point. Regardless

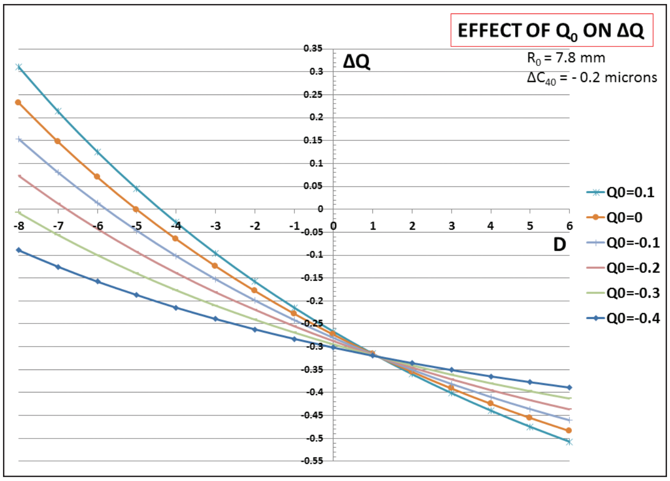

Figure 4. Effect of the preoperative corneal asphericity (Q_0) on the change in asphericity (ΔQ) to modify the spherical aberration coefficient C_{40} by -0.2 μm on a 6-mm zone for different magnitudes of defocus correction (-8 to +6 diopters [D]) (y axis).

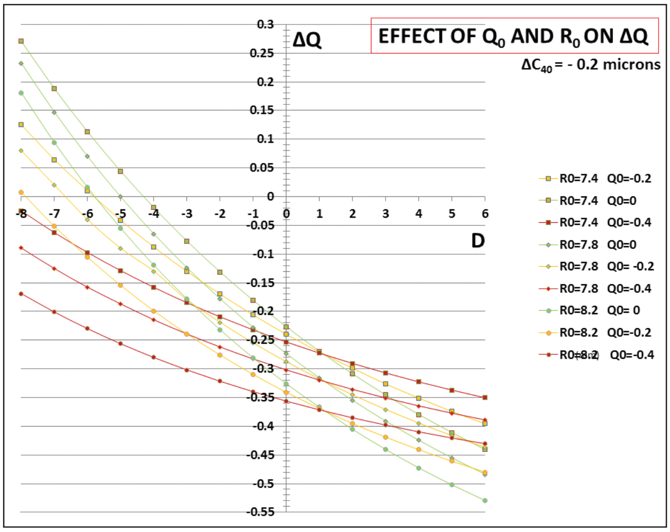

Figure 5. Effects of the preoperative apical curvature and asphericity on the change in the corneal asphericity ΔQ (x axis) to produce -0.2 μm change in spherical aberration for corrections comprised between -8 and +6 diopters (D).

of the preoperative Q value, the required change in the Q value is approximately -0.32. Beyond +1 D of correction, the more oblate the initial profile, the larger the required change of asphericity toward increased prolateness.

Figure 5 shows that the combined effect of the preoperative apical radius (R_0) and asphericity (Q_0) changes on the target change in the Q value (ΔQ) for the same intended variation in the corneal SA (ΔC^4_0 = -0.2 µm) and defocus corrections comprised between -8 and +6 D. The overlap between the curves suggests that both the apical radius and initial asphericity have to be taken into consideration to estimate accurately the required change in corneal asphericity, for the same planned change in the spherical aberration (ΔZ^4_0). This trend seems less pronounced for low myopic and hyperopic defocus values.

Combined Influence of the Change in Corneal Spherical Aberration (ΔC^4_0), Initial Corneal Asphericity (Q_0), and Paraxial Correction (D) on ΔQ

Figure 6 allows visualization of the relations between ΔQ and ΔC^4_0 for corrections ranging between -8 and +6 D, values of ΔC^4_0 of 0, -0.2, and 0.4 µm, and three different values of Q_0 (from +0.1 to -0.4, by 0.1 steps). For the same magnitude of the paraxial correction, the larger the magnitudes of the required change in spherical aberration (ΔC^4_0), the larger the influence of the initial corneal asphericity. The more pronounced the required change in the corneal spherical aberration and the more prolate the preoperative Q value, the less pronounced the influence of the magnitude of the paraxial correction. Interestingly, for an initial corneal asphericity value of Q = -0.4, to change the spherical aberration by an amount

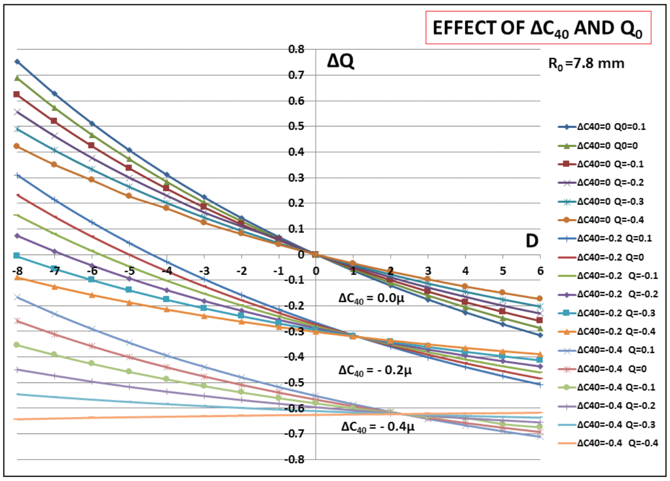

Figure 6. Required changes in asphericity ΔQ (x axis) for various corrections (-8 to +6 diopters [D]) (y axis) and three magnitudes of change in the corneal spherical aberration (6-mm zone): ΔC^4_0 = 0, -0.2, and -0.4 (μm).

of ΔC^4_0 = -0.40 µm, the required change in the corneal asphericity (ΔQ) is almost constant (ΔQ = -0.60) regardless of the value of the defocus correction.

For a specific change in spherical aberration (ΔC^4_0), there is one defocus correction value for which the influence of the initial asphericity becomes insignificant: D = 0 D for ΔC^4_0 = 0 µm, D ~ +1 D for ΔC^4_0 = -0.2 µm, and D ~ +2 D for ΔC^4_0 = -0.4 µm.

DISCUSSION

Despite its shortcomings, modeling the anterior corneal shape in cross section is a useful approximation, which provides a simple and "intelligible" model in clinical terms.[25-31] New software and laser technologies, such as **AQ3**Custom Q (Q-factor-optimized) ablation profiles, allow customized treatments that can either aim at correcting higher-order aberrations[32,33] or plan the induction of spherical aberrations.[35] However, the expected contribution to the variation in spherical aberration (ΔC^4_0) due to the induction of a new apical curvature and asphericity (Q-value) have not been fully explored.

We have evaluated the theoretical relationships between corneal asphericity modification, ΔQ, and the change in corneal spherical aberration, ΔC^4_0, for various negative and positive defocus correction (myopic and hyperopic treatment) values and combinations of preoperative corneal asphericities, Q_0, and preoperative apical radii of curvature, R_0. We are not aware of previous studies aimed at studying these parameters in a refractive surgical situation. We used inner-products between the conic and Zernike functions to correlate conic parameters (R, Q) with Zernike coefficients ($C[_{j,0}]$). In a ray-tracing study of the relations between corneal asphericity and longitudinal spherical aberration, Calossi calculated the asphericity necessary to maintain physiological corneal longitudinal spherical aberration (as a function of the spherical equivalent corrected with photoablative surgery) and found that the flattening of the cornea reduced longitudinal spherical aberration.[17] We observed a similar trend, but our predicted change in Q-values slightly differed from those calculated by Calossi. This is not unexpected because spherical aberration, expressed by the Zernike coefficient Z^4_0, differs from longitudinal spherical aberration, expressed in diopters.

Our results demonstrate that the corneal preoperative asphericity influences significantly the value of the required change in asphericity to achieve the desired variation in C^4_0, especially for high magnitudes of myopic or hyperopic corrections. Arba Mosquera and de Ortueta used Taylor's polynomial expansion to obtain equations that were deduced by identification in a term-by-term of same radial order fashion.[19] Because Taylor's expansion provides a local approximation in which accuracy is highest in a narrow domain close to the center of the analyzed zone (ie, the vertex of the corneal surface), the relation between C^4_0 and Q provided by Arba Mosquera and de Ortueta is linear. Our data, which were obtained by inner product on a 6-mm zone, clearly suggest that the influence of the preoperative asphericity on the required change in asphericity is in fact not linear (**Figure 3**).

Using ray-tracing techniques on a **AQ3**Navarro eye model, Amigó et al. studied the effect of the corneal asphericity and pupil aperture on the refractive status of the eye.[35] The methodology used was wavefront refraction, based on a definition of refractive state because the power of the correcting lens required to optimize the eye's optical quality. A variety of metrics for wavefront quality and retinal image quality were used for this computation. The relation between the spherical aberration coefficient value (C^4_0) and the asphericity of the corneal profile was computed using Taylor's expansion and identification of the terms of similar degree. They showed that the modification of the corneal asphericity, keeping the same apical radius (equivalent of no paraxial defocus change), resulted in a change in spherical aberration of magnitudes similar to those predicted by our model. They also demonstrated that the modification of the spherical aberration would change the refractive status of the eye and that this change was influenced by the radius of the pupil.

Determining the optimal change in fourth-order spherical aberration was outside the scope of our study; we limited the scope of our study to studies of the required changes of corneal asphericity needed to modify various amounts of fourth-order spherical aberration. Of note is that the desirable changes in fourth-order spherical aberration would have to be determined individually, based on the patient's ocular physiological parameters (such as pupil diameter and pupil dynamics) and the patient's visual needs.[4,35] The results of our study show that the refractive surgeon should consider several parameters that influence the magnitude of the needed change in the corneal asphericity (ΔQ) to obtain a specific target change in spherical aberration (ΔC^4_0). Furthermore, our calculations demonstrate that ΔQ does not have to be negative to induce negative ΔC^4_0; for certain values of preoperative apical radii and corneal asphericities, the required change in corneal asphericity is positive (induction of less prolate or more oblate postoperative corneal profile), despite targeting an increase in negative spherical aberration. This is explained by the fact that central corneal flattening in myopic corrections contributes to the reduction of positive, or induction of negative, spherical aberration. For laser refractive corrections of high myopic errors, an increase of negative spherical aberration often requires an increase of corneal prolateness, but not always. Under certain conditions, it can require a change in ΔQ toward increased oblateness: the corneal profile may have to become slightly oblate to allow for the control of the spherical aberration after high myopic ablations.

In previous studies of large myopic corrections, large postoperative increases in positive spherical aberration have been reported after myopic LASIK.[21] These increases in positive spherical aberration are not consistent with the predictions by our analysis. We hypothesize that these findings reflect an achieved optical zone smaller than the eye's entrance pupil,[37,39] rather than the nonaspheric characteristic of these early myopic profiles of ablation. Simulations of changes in efficiency due to reflection and non-normal incidence of the laser light have shown a further increase in corneal asphericity,[39] which has been verified experimentally with various excimer laser platforms.[18] Recently, the laser ablation algorithms have benefited from the application of correction factors for efficiency effects and have been optimized to reduce the induction of spherical aberration. A recent study aimed at studying the effect on wavefront aberrations of wavefront-optimized femtosecond laser-assisted LASIK using the WaveLight Allegretto Wave Eye-Q (**AQ3**Alcon Laboratories, Inc.) reported a small increase in spherical aberration of 0.03 ± 0.02 µm for low myopia, but decreases were observed for moderate and high myopia, with no significant induction of spherical aberration at all refractive errors.[40]

All femtosecond laser refractive lenticule extractions are not affected by the loss of excimer energy in the corneal periphery, and our predictions may be more congruent with these techniques than those based on the use of the excimer laser for altering the corneal contour.[41]

Our study has important clinical implications during excimer laser keratorefractive surgery for both emmetropization and presbyopic compensation. The optimal postoperative spherical aberration values (ΔC^0_4) are different in these situations. Once the optimal postoperative spherical aberration value (ΔC^4_0) is determined for a particular patient, our data could be used to determine the required change in the corneal asphericity (ΔQ). In the case of paraxial emmetropization, inducing some level of ocular negative spherical aberration incurred by peripheral corneal flattening would induce some hyperopic defocus for non-paraxial rays. In the context of presbyopia compensation, the increase in negative ocular spherical aberration may become valid only if the paraxial defocus correction (controlled by the change in the apical radius of curvature of the cornea in our model) aims at some level of negative (myopic) defocus. Of importance is the fact that for hyperopic corrections and large intended changes in spherical aberration (ΔC^4_0 = -0.4 µm), the required change in asphericity is comprised between ΔQ = -0.55 and -0.7, depending on the preoperative corneal asphericity and paraxial defocus correction (**Figure 6**). This may help to simplify further custom-aspheric nomogram development.

An excessive increase in spherical aberration has shown to induce halo vision and/or blurring, especially under scotopic conditions where pupils are large. In general, it seems that small values of spherical aberration commonly found in eyes decrease acuity by a relatively small amount under optimum-focus conditions. Using ray tracing in a Navarro eye model, Amigó et al. have shown that, for 6-mm pupils, exceeding a more negative spherical aberration than -0.4 µm (corresponding to a Q value more negative than -1.25 in their model) does not increase the amount of accommodation by pupil miosis.[35]

We limited our calculations to the corneal plane: the net contribution of the change in spherical aberration of corneal origin to the ocular wavefront may be slightly different. An entrance pupil of 6 mm at the corneal plane would require the physical pupil diameter to be slightly lower (by approximately 20%) to avoid the introduction of spherical aberration due to insufficient coverage of the pupil. Also, some physical constraints, such as the "cosine effect," should be controlled,[18,39] along with that of the biomechanical and wound healing response. The final retinal image is influenced mainly, but not exclusively, by the anterior corneal wave aberration. We limited our analysis to the fourth-order spherical aberration, which has often been considered the main aberration of the cornea.[42-44] Preoperative consideration of corneal aberrations alone would result in incorrect assumptions for the optical performance of the eye.[45,46]

Our study focused on the expected variation in the fourth-order corneal spherical aberration coefficient induced by reshaping the anterior corneal surface. We did not consider the posterior surface of the cornea in this study, because it contributes a relatively small fraction of the eye power. Although the response of the posterior corneal surface to myopic LASIK may vary with different ablation depths,[47] the degree of change is relatively low,[48,49] with a tendency to return to preoperative levels 1 to 3 months postoperatively. Hence, our theoretical conclusions may remain valid in the presence of posterior corneal shape changes after surgery. However, our theoretical conclusions may not be applicable to wavefront-guided ablations, which are based on a different approach than customized Q-value ablations and which include the correction of high-order aberrations and incur parabolic shape for lower-order aberrations.

We have established clinically relevant theoretical relationships between the change in corneal shape parameters and the resultant variation of the corneal spherical aberration. Due to the simplicity of our model, our results may be considered as an approximation that may be helpful to the corneal refractive surgeon when estimating the expected change in the corneal wavefront fourth-order spherical aberration in response to attempted changes of asphericity. These findings could be useful to better understand and further refine our surgical approaches aimed at customizing the postoperative Q-value of the cornea and potentially increase the depth of focus of the eye in ametropias and presbyopia.

AUTHOR CONTRIBUTIONS

Study concept and design (); data collection (); analysis and interpretation of data (); drafting of the manuscript (); critical revision of the manuscript (); administrative, technical, or material support (); supervision ()

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General Please verify the formatting of all Zernike coefficients within text. Per journal style, Zernike coefficents are expressed as $X^{x}_{x.}$ Should this apply throughout text (i.e., ΔC^{4}_{0})?

References 34, 36, and 38 were not cited in text. Please cite or delete these references.

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APPENDIX A

Analytical Expression of a Conic Section

The analytical expression of the profile of conic section of apical radius R and asphericity coefficient Q is:

$$z = f(\rho,R,Q) = R - \sqrt{\frac{(R^2-(1+Q)\rho^2)}{1+Q}} \text{ if } Q \neq -1 \qquad (1)$$

$$z = \frac{\rho^2}{2R} \text{ if } Q = -1 \qquad (2)$$

The partial derivatives $\frac{\partial f}{\partial R}$ and $\frac{\partial f}{\partial Q}$ are given by the formulas:

$$\frac{\partial f}{\partial R}(\rho,R,Q) = -\frac{f(\rho,R,Q)}{\sqrt{R^2-(1+Q)\rho^2)}} \text{ if } Q \neq -1 \qquad (3)$$

$$= \frac{-1}{2R^2}\rho^2 \text{ if } Q = -1 \qquad (4)$$

$$\frac{\partial f}{\partial Q}\left(\rho,R,Q\right) = -\frac{f(\rho,R,Q)}{1+Q} + \frac{\rho^2}{2(1+Q)\sqrt{R^2-(1+Q)\rho^2}} \text{ if } Q \neq -1 \qquad (5)$$

$$= \frac{1}{8R^3}\rho^4 \text{ if } Q = -1 \qquad (6)$$

Zernike Functions for Fitting the Corneal Surface

Because of the rotationally symmetry of our model, the Zernike fitting can be reduced to:

$z = f(\rho,R,Q) = \Sigma_{p=0}^{\infty} c_{2p}^0(R,Q) R_{2p}^0(\rho)$ where R_{2p}^0 correspond to the Zernike radial polynomials given by:

$$R_{2p}^0(\rho) = \sqrt{2p+1}\,\Sigma_{k=0}^{p}(-1)^k \frac{(2p-k)!}{k!\,((p-k)!)^2}\rho^{2\rho-2k} \text{ with } 0 < \rho < 1 \qquad (7)$$

The coefficients of the Zernike R_{2p}^0 (R, Q) can be obtained by the following:

$$c_{2p}^0(R,Q) = \int_0^1 f(\rho,R,Q) R_{2p}^0(\rho)\, 2\rho\, d\rho \qquad (8)$$

The polynomials of degree 2p ≤ 6 are given by:

$$R_0^0(\rho) = 1 \qquad (9)$$

$$R_2^0(\rho) = \sqrt{3}\,(2\rho^2 - 1) \qquad (10)$$

$$R_4^0(\rho) = \sqrt{5}\,(6\rho^4 - 6\rho^2 + 1) \qquad (11)$$

$$R_6^0(\rho) = \sqrt{7}\,(20\rho^6 - 30\rho^4 + 12\rho^2 - 1) \qquad (12)$$

Effect of Parameters Variations on the Zernike Coefficients

The partial derivatives from R and Q, of the c_{2p}^0 coefficients $\left(\frac{\partial c_{2p}^0}{\partial R} \text{ and } \frac{\partial c_{2p}^0}{\partial R}\right)$ are given by:

$$\frac{\partial c_{2p}^{0}}{\partial R}(R,Q) = \int_0^1 \frac{\partial f}{\partial R}(\rho,R,Q)\,R_{2p}^{0}(\rho)\,2\rho\,d\rho \qquad (13)$$

$$\frac{\partial c_{2p}^{0}}{\partial Q}(R,Q) = \int_0^1 \frac{\partial f}{\partial Q}(\rho,R,Q)\,R_{2p}^{0}(\rho)\,2\rho\,d\rho \qquad (14)$$

Hence:

$$\frac{\partial c_{2p}^{0}}{\partial R}(R,Q) = -\int_0^1 \frac{f(\rho,R,Q)}{\sqrt{R^2-(1+Q)\rho^2}}\,R_{2p}^{0}(\rho)\,2\rho\,d\rho \quad \text{if Q} \neq -1 \qquad (15)$$

$$= -\frac{1}{2R^2}\int_0^1 \rho^2 R_{2p}^{0}(\rho)\,2\rho\,d\rho \quad \text{if Q} = -1 \qquad (16)$$

$$\frac{\partial c_{2p}^{0}}{\partial Q}(R,Q) = \frac{1}{1+Q}\left(-c_{2p}^{0}(R,Q) + \int_0^1 \frac{\rho^2}{2\sqrt{R^2-(1+Q)\rho^2}}R_{2p}^{0}(\rho)\,2\rho\,d\rho\right) \quad \text{if Q} \neq -1 \qquad (17)$$

$$= -\frac{1}{8R^3}\int_0^1 \rho^4 R_{2p}^{0}(\rho)\,2\rho\,d\rho \quad \text{if Q} = -1 \qquad (18)$$

These partial derivatives are not amenable to an analytic calculation, but can allow to measure the sensitivity of the Zernike coefficients to the variations of R and Q (ΔR and ΔQ) of the corneal surface:

$$\Delta c_{2p}^{0} \approx \frac{\partial c_{2p}^{0}}{\partial R}(R,Q)\times\Delta R + \frac{\partial c_{2p}^{0}}{\partial Q}(R,Q)\times\Delta Q \qquad (19)$$

By knowing the values of the initial parameters R0 and Q0, it is possible to estimate the value of Q1 which, for a small variation of the apical radius R from R0, would induce a desirable variation Δc_{2p}^{0} of a Zernike coefficient:

$$Q \approx Q_0 + \frac{\Delta c_{2p}^{0} - \frac{\partial c_{2p}^{0}}{\partial R}(R_0,Q_0)\times(R_1-R_0)}{\frac{\partial c_{2p}^{0}}{\partial Q}(R_0,Q_0)} \qquad (20)$$

This can be solved numerically in two steps:

$$Q_1^{*} \approx Q_0 + \frac{\Delta c_{2p}^{0} - (c_{2p}^{0}(R_1,Q_0) - c_{2p}^{0}(R_0,Q_0))}{\frac{\partial c_{2}^{0}}{\partial Q}\left(\frac{R_0+R_1}{2},Q_0\right)} \qquad (21)$$

And then:

$$Q_1^{**} \approx Q_1^{*} + \frac{\Delta c_{2p}^{0} + c_{2p}^{0}(R_0,Q_0) - c_{2p}^{0}(R_1,Q_1^{*})}{\frac{\partial c_{2}^{0}}{\partial Q}(R_1,Q_1^{*})} \qquad (22)$$

These calculations were performed with Maple 11 (Maplesoft, Waterloo, ON, Canada) and numerical values were entered for a range of different dioptric treatments, initial asphericities, and radii of curvature, to estimate the value of the required postoperative asphericity change ($\Delta Q = \Delta Q_1^{**} - Q_0$) to induce the change in the fourth order Zernike spherical aberration C_4^0. For each calculation, the obtained value of ΔQ was used to compare analytically the value of the achieved vs target ΔC_4^0. In all our numerical calculations, this difference was less than 0.5%.