Appendix A

A.1.1Determination of the F function

Let us consider a pseudophakic model eye with a cornea and IOL, modeled as thick lenses of respective total powers K and P (Figure A1).

The total power of the eye (De) is given by:

$$D_e = K + P - \frac{K\ P\ ELP_T}{n_a} = \frac{n_v}{H'_e F'_e} \quad (Eq. A1)$$

where ELP_T refers to the distance between the cornea's principal image plane and the IOL's principal object plane, $H'_eF'_e$ corresponds to the distance separating the eye's principal image plane H'_e from the entire eye's focal point F'_e when the effective lens position is at ELP. Here, n_a is the refractive index of the aqueous, n_v is the refractive index of the vitreous.

When an amount of ΔELP_T shifts the ELP_T, the total eye power becomes:

$$\widetilde{D_e} = K + P - \frac{K\ P\ (ELP_T + \Delta ELP_T)}{n_a} = \frac{n_v}{\widetilde{H'_e}\widetilde{F'_e}} \quad (Eq. A2)$$

where $\widetilde{H'_e}\widetilde{F'_e}$ corresponds to the distance separating the eye's principal image plane from the entire eye's focal point when the effective lens position is at $ELP_T + \Delta ELP_T$,

The change in total ocular power refraction, when the IOL position shifts by ΔELP_T is given by

$$\Delta R = D_e - \widetilde{D_e} = \frac{n_v}{H'_e F'_e} - \frac{n_v}{H'_e \widetilde{F'_e}} = D_e - \frac{n_v}{H'_e F'_e + \Delta HF} \quad (Eq. A3)$$

where $\Delta HF = \widetilde{H'}_e\widetilde{F'}_e - H'_eF'_e + H'_e\widetilde{H'}_e$ (Figure A1).

We can rewrite Eq. A3 as follows:

$$\Delta R = D_e - \left(\frac{n_v}{\frac{n_v}{D_e} + \Delta HF}\right) = D_e\left(1 - \frac{1}{1 + \frac{D_e\,\Delta HF}{n_v}}\right) \quad (Eq. A4)$$

Using the Taylor expansion of $\frac{1}{1+\frac{D_e\,\Delta HF}{n_v}} \approx 1 - \frac{D_e\,\Delta HF}{n_v}$, we obtain the following approximation:

$$\Delta R \approx \frac{{D_e}^2 \Delta HF}{n_v} \quad (Eq. A5)$$

The distance from the principal image plane of the cornea to F'e in the thick pseudophakic eye is given by [3] :

$$L_T = ELP_T + \frac{n_v - \frac{n_v}{n_a} KELP_T}{K + P - \frac{KPELP_T}{n_a}} \quad (Eq\ A6)$$

With the assumption that $n_a = n_v$, the derivation of L_T by ELP_T yields

$$\frac{\Delta L_T}{\Delta ELP_T} = 1 - \frac{1}{\left(\frac{PELP_T}{n_a} - \frac{P}{K} - 1\right)^2} \quad (Eq\ A7)$$

which is equivalent to:

$$\frac{\Delta L_T}{\Delta ELP_T} = 1 - \frac{K^2}{{D_e}^2} \quad (Eq\ A8)$$

Hence:

$$\Delta L_T = \left(1 - \frac{K^2}{{D_e}^2}\right) \Delta ELP_T \quad (Eq\ A9)$$

Using Eq.A5 we get:

$$\Delta R \approx \frac{\Delta HF}{\Delta L_T} \frac{{D_e}^2}{n_v} \left(1 - \frac{K^2}{{D_e}^2}\right) \Delta ELP_T \quad (Eq. A10)$$

Hence, we obtain:

$$\Delta R \approx \frac{\Delta HF}{\Delta L_T} \frac{1}{n_v} \left({D_e}^2 - K^2\right) \Delta ELP_T \quad (EqA11)$$

If we consider ELP_T small, then $D_e \approx K + P$.

We finally get:

$$\Delta R \approx \frac{\Delta HF}{n_v \Delta L_T} \left(P^2 + 2KP\right) \Delta ELP_T \quad (Eq. A12)$$

Therefore:

$$F(K,P) \approx C\left(P^2 + 2KP\right) \quad (Eq.A13)$$

where $C = \frac{\Delta HF}{n_v \Delta L_T}$

Using Equation A12 we obtain :

$$\Delta ELP_T \approx \frac{1}{C(P^2 + 2KP)} \Delta R \quad (Eq.A14)$$

Finally, the ELP increment aimed at canceling the mean PE on the whole set can be written as:

$$\Delta ELP_T \approx -\frac{\bar{E}}{C\overline{(P^2 + 2KP)}} \quad (Eq.A15)$$

where $\overline{(P^2 + 2KP)} = \frac{\sum_{i=1}^{N} ({P_i}^2 + 2K_i, P_i)}{N}$ is the mean of $(P^2 + 2KP)$ on the whole dataset, K_i and P_i are the corneal and IOL powers of the i^{th} eye of the data set of N eyes.

A.1.2 Estimation of the linear regression coefficient from the theoretical eye model

We used a pseudophakic thick lens eye model to estimate the numerical value of C (Figure A2).

One can calculate the theoretical impact of a variation of the ELP_T (ΔELP_T) on the refraction of the thick lens pseudophakic eye (ΔR) using the vergence formula [3] :

$$\Delta R = -\frac{1}{\frac{1}{\frac{n_a}{\frac{n_a}{-\frac{n_v}{\left(L_T - (ELP_T + \Delta ELP_T)\right)} + \mathrm{P}} - (ELP_T + \Delta ELP_T)} + K} - d} - R \quad (Eq.A16)$$

, where n_a and n_v are the refractive indices of the aqueous humor and vitreous, respectively, and P and K are the IOL and total corneal power, respectively. L_T is the axial length extending from the cornea's image principal plane (PP) to the photoreceptor plane minus the distance between the IOLs' principal planes. ELP_T is the effective lens position from the image PP of the cornea to the object PP of the IOL plane. d is the distance to the spectacle plane. R is the SE refraction at the spectacle plane. This equation enables us to compute the impact on R, of a given variation in ELP_T (ΔELP_T), analogous to a change in lens constant.

We limited our simulations to 385 (11x35) different modeled eyes of total corneal power varying between 38D and 48D, receiving IOLs of power ranging between 1D and 35D. The IOLs were modeled as symmetrical biconvex lenses (null Coddington shape factor). The distance to the spectacle plane was set to d = 12 mm. All modeled eyes were emmetropic for their initial ELP, corneal and IOL powers, and axial length. The baseline numerical IOL-related and biometric values used for the numeric simulations are listed in Table A1. The refractive index values used for numerical simulations were: n_a=1.336 (aqueous), n_v=1.336 (vitreous), n_c=1.376 (corneal stroma), and n_i=1.45 (IOL material).

The refraction change at the spectacle plane (ΔR) was computed for +1 mm and -1 mm incremental variations in the ELP_T in a series of modeled eyes of various corneal and IOL powers. Linear regression against the expression $(P^2 + 2KP)$ was performed for each of the tested incremental ELP variations to obtain a numerical value of the linear regression coefficient C.

Figure A3 represents the plot of the regression between (P^2+2KP) and ΔR for shifts in ELP of +1mm and -1 mm, and the results of the linear regression.

The values of the linear regression coefficients through the origin were 0.0006+/-0.66 10^{-7} (ΔELP=1mm) and -0.0006+/-0.59 10^{-7} (ΔELP=1mm).

Figure A1

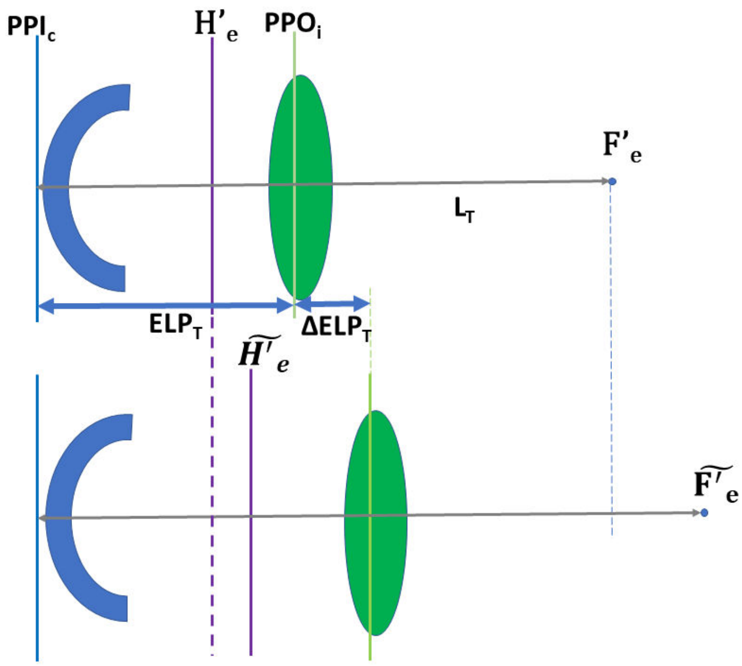

Figure A1: Schematic representation of the shift of the IOL position

Figure A2

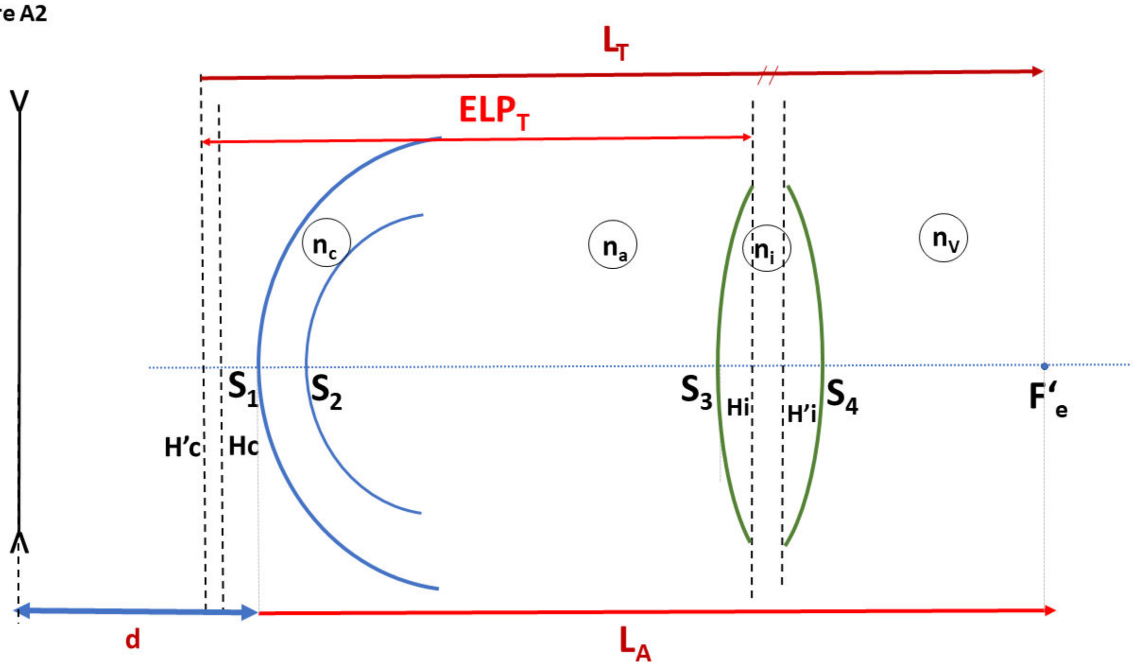

Figure A2: Paraxial, four-surface, thick-lens, pseudophakic eye model. F'_e is the back focal point of the paraxial schematic pseudophakic eye. L_A is the anatomic axial length from the anterior surface of the cornea to the photoreceptor plane (PP). L_T is the thick lens axial length, which connects the principal image plane of the cornea H'_c to the PP and is reduced by the distance $H_iH'_i$ separating the two principal planes of the IOL. ELP_T is the effective thick lens position joining the cornea's principal image plane to the IOL's principal object plane d is the distance to the spectacle plane.

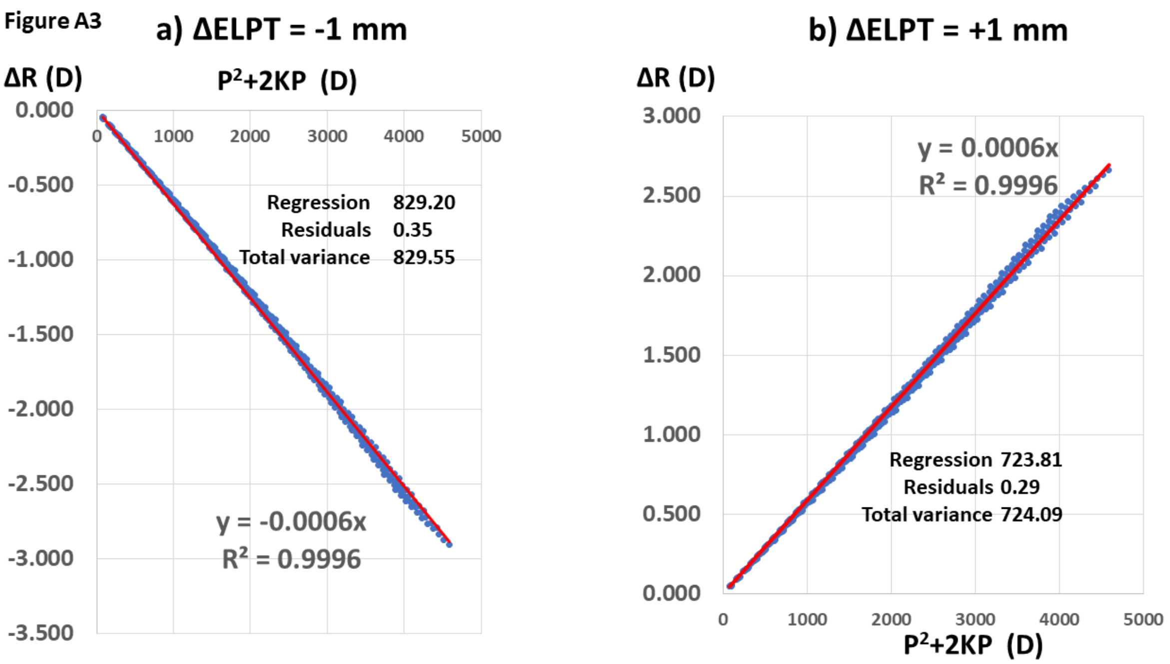

Figure A3: Regression plots between P^2+2KP and ΔR for the 385 modeled pseudophakic eyes.

Table A1: Distribution of parameters used for the modeling of thick lens pseudophakic eyes. P: IOL power (D), S1S3: distance between the anterior surface of the cornea and the anterior surface of the implant (mm). K: total corneal power (D), L: Anatomical axial length (mm)

		←-- K --→										
		38	39	40	41	42	43	44	45	46	47	48
P	S1S3	←--L--→										
1	6.50	34.54	33.67	32.85	32.07	31.32	30.61	29.92	29.27	28.64	28.04	27.46
2	6.42	33.96	33.13	32.34	31.59	30.87	30.18	29.52	28.89	28.28	27.70	27.14
3	6.34	33.41	32.61	31.85	31.12	30.43	29.77	29.13	28.52	27.93	27.37	26.82
4	6.26	32.87	32.10	31.37	30.67	30.00	29.36	28.75	28.15	27.59	27.04	26.51
5	6.18	32.34	31.61	30.90	30.23	29.58	28.96	28.37	27.80	27.25	26.72	26.21
6	6.10	31.83	31.13	30.45	29.80	29.18	28.58	28.00	27.45	26.92	26.40	25.91
7	6.02	31.34	30.66	30.00	29.38	28.78	28.20	27.64	27.11	26.59	26.09	25.61
8	5.94	30.86	30.20	29.57	28.97	28.39	27.83	27.29	26.77	26.27	25.79	25.32
9	5.86	30.39	29.76	29.15	28.57	28.00	27.46	26.94	26.44	25.96	25.49	25.03
10	5.78	29.93	29.32	28.74	28.17	27.63	27.11	26.60	26.12	25.65	25.19	24.75
11	5.7	29.49	28.90	28.34	27.79	27.27	26.76	26.27	25.80	25.34	24.90	24.48
12	5.62	29.06	28.49	27.94	27.42	26.91	26.42	25.95	25.49	25.05	24.62	24.20
13	5.54	28.64	28.09	27.56	27.05	26.56	26.09	25.63	25.18	24.75	24.34	23.93
14	5.46	28.23	27.70	27.19	26.69	26.22	25.76	25.31	24.88	24.47	24.06	23.67
15	5.38	27.83	27.31	26.82	26.34	25.88	25.44	25.00	24.59	24.18	23.79	23.41
16	5.3	27.43	26.94	26.46	26.00	25.55	25.12	24.70	24.30	23.90	23.52	23.15
17	5.22	27.05	26.57	26.11	25.66	25.23	24.81	24.40	24.01	23.63	23.26	22.90
18	5.14	26.68	26.21	25.77	25.33	24.91	24.51	24.11	23.73	23.36	23.00	22.65
19	5.06	26.31	25.86	25.43	25.01	24.60	24.21	23.83	23.45	23.09	22.74	22.40

20	**4.98**	**25.95**	**25.52**	**25.10**	**24.69**	**24.30**	**23.91**	**23.54**	**23.18**	**22.83**	**22.49**	**22.16**
21	**4.9**	**25.60**	**25.18**	**24.78**	**24.38**	**24.00**	**23.63**	**23.27**	**22.91**	**22.57**	**22.24**	**21.92**
22	**4.82**	**25.26**	**24.85**	**24.46**	**24.08**	**23.70**	**23.34**	**22.99**	**22.65**	**22.32**	**22.00**	**21.68**
23	**4.74**	**24.93**	**24.53**	**24.15**	**23.78**	**23.41**	**23.06**	**22.72**	**22.39**	**22.07**	**21.75**	**21.45**
24	**4.66**	**24.60**	**24.21**	**23.84**	**23.48**	**23.13**	**22.79**	**22.46**	**22.14**	**21.82**	**21.52**	**21.22**
25	**4.58**	**24.28**	**23.90**	**23.54**	**23.19**	**22.85**	**22.52**	**22.20**	**21.88**	**21.58**	**21.28**	**20.99**
26	**4.5**	**23.96**	**23.60**	**23.25**	**22.91**	**22.58**	**22.26**	**21.94**	**21.64**	**21.34**	**21.05**	**20.76**
27	**4.42**	**23.65**	**23.30**	**22.96**	**22.63**	**22.31**	**21.99**	**21.69**	**21.39**	**21.10**	**20.82**	**20.54**
28	**4.34**	**23.35**	**23.01**	**22.68**	**22.35**	**22.04**	**21.74**	**21.44**	**21.15**	**20.87**	**20.59**	**20.32**
29	**4.26**	**23.05**	**22.72**	**22.40**	**22.08**	**21.78**	**21.48**	**21.19**	**20.91**	**20.64**	**20.37**	**20.11**
30	**4.18**	**22.76**	**22.43**	**22.12**	**21.82**	**21.52**	**21.23**	**20.95**	**20.68**	**20.41**	**20.15**	**19.89**
31	**4.1**	**22.47**	**22.16**	**21.85**	**21.56**	**21.27**	**20.99**	**20.71**	**20.45**	**20.19**	**19.93**	**19.68**
32	**4.02**	**22.19**	**21.88**	**21.59**	**21.30**	**21.02**	**20.75**	**20.48**	**20.22**	**19.96**	**19.71**	**19.47**
33	**3.94**	**21.91**	**21.61**	**21.33**	**21.05**	**20.77**	**20.51**	**20.25**	**19.99**	**19.74**	**19.50**	**19.26**
34	**3.86**	**21.63**	**21.35**	**21.07**	**20.80**	**20.53**	**20.27**	**20.02**	**19.77**	**19.53**	**19.29**	**19.06**
35	**3.78**	**21.37**	**21.09**	**20.82**	**20.55**	**20.29**	**20.04**	**19.79**	**19.55**	**19.31**	**19.08**	**18.86**